The 3rd International Workshop on Matrix Analysis and Applications

1. 출발 전 유의사항

(1) 단기연수 신청서 제출 (invoice, 초청장, 논문초록 첨부)

(2) 여권발급(수원월드컵 경기장 주차장 입구)

(3) 비자신청, 티켓예약(여권영어이름과 동일), 호텔예약

(4) 환전(학교 내 우리은행)

(5) 도착지 기후 확인

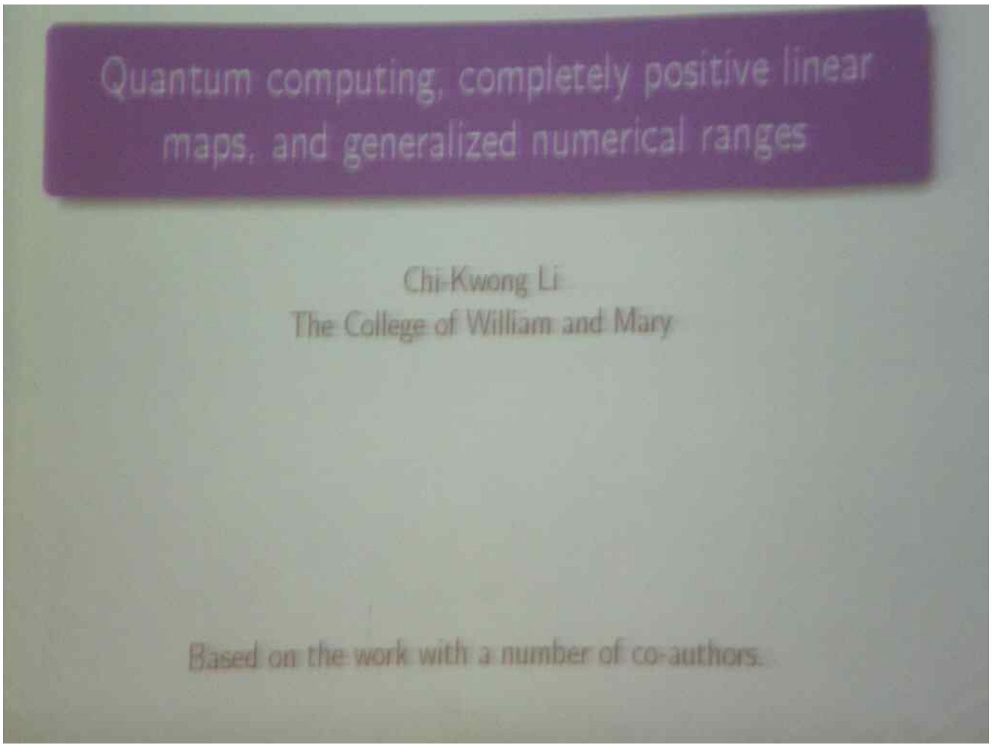
(6) 음식문제(중국은 모든 음식에 향신료를 사용함)

(7) 전자제품(110volt), 콘센트 모양이 틀림

2. 귀국 후 유의사항

10일 이내 연수결과 및 귀국보고서 제출
(출입국 사실 확인용 여권사본, 왕복 항공 탑승권 원본, 참가 학술회의 PROCEEDINGS 해당 부분 사본)


第三次国际矩阵分析与应用研讨会
The Third International Workshop on Matrix Analysis and Applications
主办：
国际线性代数协会
国际自然科学基金委
浙江省自然科学基金委
Sponsored by:
International Linear Algebra Society
National Natural Science Foundation of China
Natural Science Foundation of Zhejiang Province
承办：浙江林学院
Hosted by: Zhejiang Forestry University


Quantum computing, completely positive linear maps, and generalized numerical ranges
Chi-Kwong Li
The College of William and Mary
Based on the work with a number of co-authors.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Weighted generating trees and Related Combinatorial Interpretations

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10. 15. 2009

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1. Weighted generating tree

A *generating tree* (GT) [Chung, 1978] is a rooted labelled tree s.t. the labels of children of each node x can be determined from the label of x itself. Thus any particular generating tree may be specified by a *succession rule* consisting of

(1) the label of the root;

(2) a rule explaining how to derive the number of children and their labels.

Let us consider a succession rule given by

$$\Omega : \begin{cases} root : (c) & \\ rule : (k) & \mapsto (c)^{z_{k-c}}(c+1)^{a_{k-c}} \cdots (k+1)^{a_0} \end{cases} \tag{1}$$

where $c \in N$, a_i, $z_i \in N_0$, $\forall i \geq 0$, $a_0 \neq 0$.

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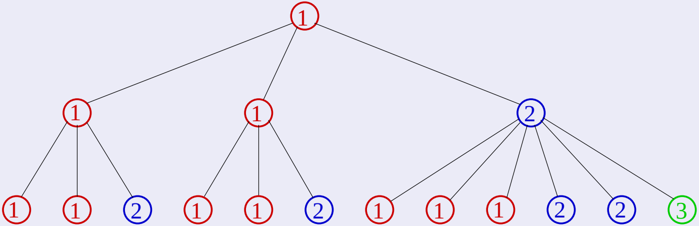
where $c \in N$, $a_i, z_i \in N_0, \forall i \geq 0, a_0 \neq 0$.

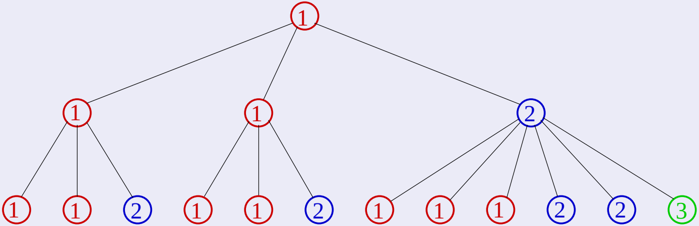
Deutsch, Ferrari and Rinaldi(2005)

- production matrix P whose (n, k)-entry is the number of nodes $(k+c)$ produced by node $(n+c)$,
- ECO matrix A_P induced by P whose (n, k)-entry is the number of nodes $(k+c)$ at level n.

Ex 1.1 Ω**:** $\begin{cases} root: & (1) \\ rule: & (k) \mapsto (1)^{k+1}(2)^{k}\cdots(k)^{2}(k+1)^{1} \end{cases}$



Ex 1.1 Ω**:** $\begin{cases} \textit{root}: & (1) \\ \textit{rule}: & (k) \mapsto (1)^{k+1}(2)^{k}\cdots(k)^{2}(k+1)^{1} \end{cases}$



$$P = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \\ & & \cdots & \end{bmatrix}, \quad A_P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 7 & 4 & 1 & 0 \\ & & \cdots & \end{bmatrix}$$

Let us consider a generalized succession rule given by

$$\Omega : \begin{cases} root : (c) \\ rule : (k) \quad \mapsto (c)^{z_{k-c}}(c+1)^{a_{k-c+j-1}} \cdots (k+j)^{a_0} \end{cases} \tag{2}$$

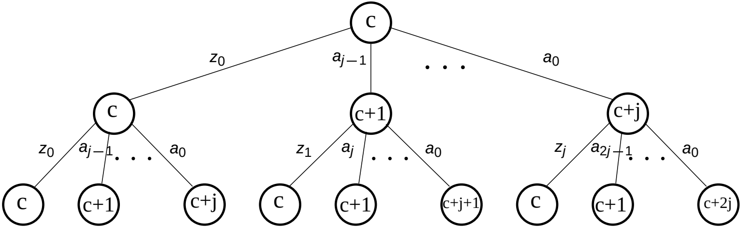
where $c \in N$, $a_i, z_i \in R$, $\forall i \geq 0$, $a_0 \neq 0$, and $j \geq 1$.

Let us consider a generalized succession rule given by

$$\Omega : \begin{cases} root : (c) \\ rule : (k) \quad \mapsto (c)^{z_{k-c}}(c+1)^{a_{k-c+j-1}} \cdots (k+j)^{a_0} \end{cases} \qquad (2)$$

where $c \in N$, $a_i, z_i \in R$, $\forall i \geq 0$, $a_0 \neq 0$, and $j \geq 1$.

We define a *weighted generating tree* (WGT) induced by Ω to be a rooted labeled tree with vertex weights and edge weights.



<The weighted generating tree>

ω-Production matrix

We define *ω-production matrix* P_ω to be a matrix whose (n,k)-entry is the weight on edge joining $(c+n)$ and $(c+k)$ produced by $(c+n)$. Thus we have

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$$P_\omega = \begin{bmatrix} z_0 & a_{j-1} & \cdots & a_0 & 0 & 0 & \cdots \\ z_1 & a_j & \cdots & a_1 & a_0 & 0 & \cdots \\ z_2 & a_{j+1} & \cdots & a_2 & a_1 & a_0 & \cdots \\ & & & \cdots & & & \end{bmatrix}.$$

ω-ECO matrix

We define ω-ECO matrix A_{P_ω} induced by P_ω to be a matrix whose (n, k)-entry is the total sum of the product of edge weights in each path from the root (c) to the node $(c + k)$ at level n. Then it can be proved that

ω-ECO matrix

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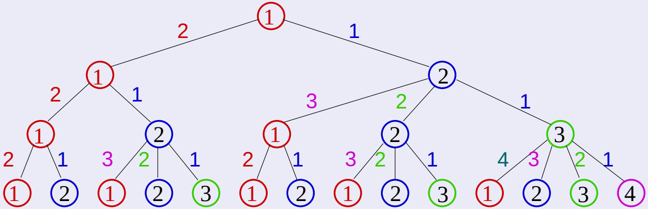
$$A_{P_\omega} = \begin{bmatrix} u^T \\ u^T P_\omega \\ u^T P_\omega^2 \\ \cdots \end{bmatrix}, \quad u^T = (1, 0, \ldots)$$

2. Total weights of paths in WGT

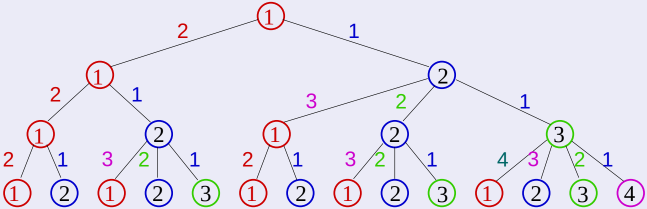
We are now interested in the total weight of paths in a WGT.

- *Weight of a path* from the root (c) to (c+k) at level n in WGT := the product of all vertex and edge weights assigned on the path.
- $q_{n,k}$:= the sum of weights of such paths.
- $Q_n := \sum_{k=0}^{n} q_{n,k}$. That is, Total weight of all the paths from the root to level n.

Example 2.1

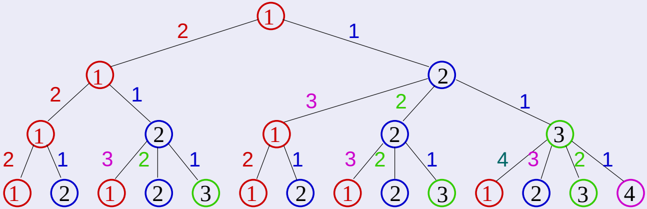


Example 2.1



$Q_0 = 1,$

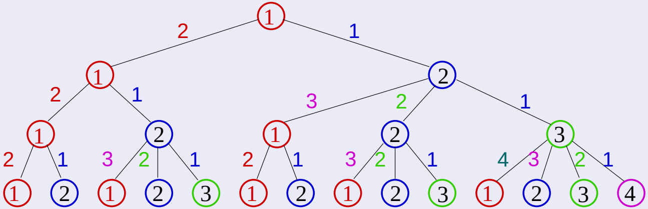
Example 2.1



$Q_0 = 1,$

$Q_1 = (1 \cdot 2 \cdot 1) + (1 \cdot 1 \cdot 2) = 4,$

Example 2.1



$$Q_0 = 1,$$
$$Q_1 = (1 \cdot 2 \cdot 1) + (1 \cdot 1 \cdot 2) = 4,$$
$$Q_2 = (1 \cdot 2 \cdot 1 \cdot 2 \cdot 1) + (1 \cdot 2 \cdot 1 \cdot 1 \cdot 2) + (1 \cdot 1 \cdot 2 \cdot 3 \cdot 1)$$
$$+ (1 \cdot 1 \cdot 2 \cdot 2 \cdot 2) + (1 \cdot 1 \cdot 2 \cdot 1 \cdot 3) = 28,$$
$\dots$

Theorem 2.2 ($GF(Q_n)$ of total weights)

Let Q_n be the total weight up to level n of the WGT.
Then the generating function for the $(Q_n)_{n\geq 0}$ is

$$\phi(t) = cu^T(I - tQ)^{-1}e$$

where

$$Q = P_\omega \cdot diag(c, c+1, \ldots),\ u^T = (1, 0, \ldots),\ e^T = (1, 1, \ldots).$$

<Note>

$$\Omega : \begin{cases} root : (c) \\ rule : (k) & \mapsto (c)^{cz_{k-c}}(c+1)^{(c+1)a_{k-c+j-1}} \cdots (k+j)^{(k+j)a_0} \end{cases}$$

Question

What is the explicit form of GF for $(Q_n)_{n\geq 0}$?

Theorem 2.3 (EGF for total weights)

Let $z_{k-1} = a_k$ in Ω with $j = 1$ and let $A(t) = \sum_{n\geq 0} a_n t^n$ ($a_0 = 1$). Then the e.g.f. for the total weight Q_n up to level n is

$$\Phi_E(t) = \frac{1}{1 - f(t)} = 1 + \sum_{n\geq 0} Q_n \frac{t^{n+1}}{(n+1)!}.$$

where $A(f(t)) = \frac{d}{dt} f(t)$ with $f(0) = 0$.

EX 2.3 $Q_n = (3n+1)!!!$

Let be the succession rule given by

$$\left\{ \begin{array}{ll} root: & (1) \\ rule: & (k) \mapsto (1)^{k+1}(2)^{k} \cdots (k)^{2}(k+1)^{1} \end{array} \right. .$$

Then $Q_n = (3n+1)!!! = 1 \cdot 4 \cdot 7 \cdots (3n+1)$.

[Solution]

$$A(t) = 1 + 2t + 3t + \cdots = \left(\frac{1}{1-t}\right)^2$$

$$\Rightarrow \left(\frac{1}{1-f}\right)^2 = \frac{df}{dt}$$

$$\Rightarrow f(t) = 1 - \sqrt[3]{1-3t}$$

$$\Rightarrow \Phi_E(t) = \frac{1}{\sqrt[3]{1-3t}} = 1 + t + 4 \cdot \frac{t^2}{2!} + 28 \cdot \frac{t^3}{3!} + \cdots .$$