Rationality problem of the Coble quartic

Sukmoon Huh

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Sungkyunkwan university

Basic notions

Throughout this talk, the base field is $\mathbb{C}$, the complex numbers.
Consider a group action

$$\nu : \mathbb{C}^* \times \mathbb{C}^{n+1} \to \mathbb{C}^{n+1}$$
$$(\lambda; z_0, \cdots, z_n) \mapsto (\lambda z_0, \cdots, \lambda z_n).$$

The complex projective space is defined to be the orbit space of the group action above;

$$\mathbb{P}_n := (\mathbb{C}^{n+1} \backslash \{0\}) / \sim .$$

If n is 1 or 2, $\mathbb{P}_n$ is called projective line or projective plane, respectively.
Let X be an algebraic variety of dimension n.

- X is called rational if there exists a birational map

$$f : X \to \mathbb{P}_n.$$

- X is called unirational if the above holds with the condition that f is dominant.

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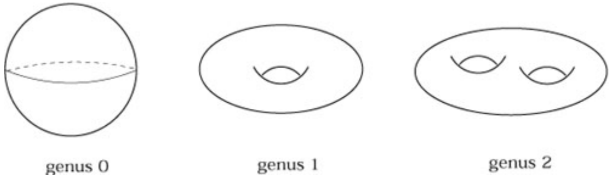
Facts

Let C be a smooth projective curve. C is characterized as a compact Riemann surface;



The projective line $\mathbb{P}_1$ is an algebraic curve of genus 0. By definition, $\mathbb{P}_1$ is rational.

- For curves and surfaces, rationality $\Leftrightarrow$ unirationality.
- (Zariski) There exists a unirational but not rational surface in characteristic $p > 0$.
- (Clemens-Griffiths) A cubic threefold is, in general, not a rational variety.
- (Conjecture) Is a cubic fourfold, in general, rational?

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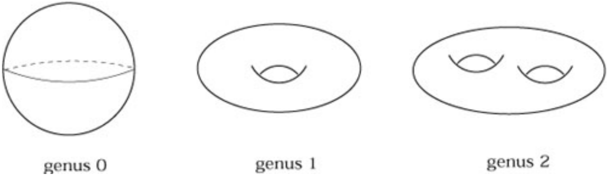

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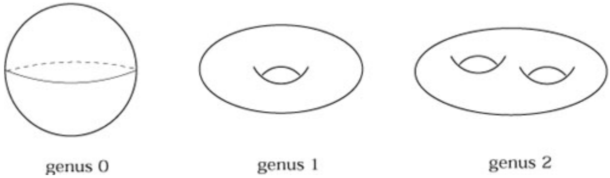

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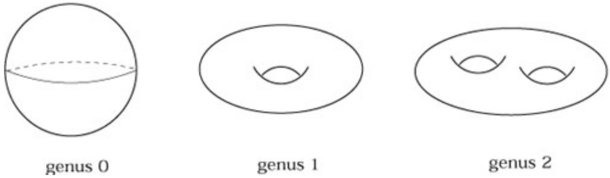

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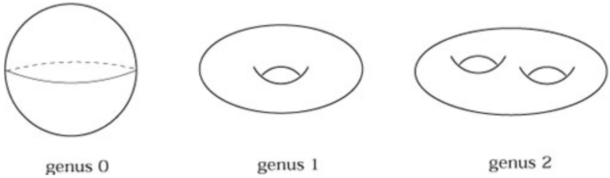

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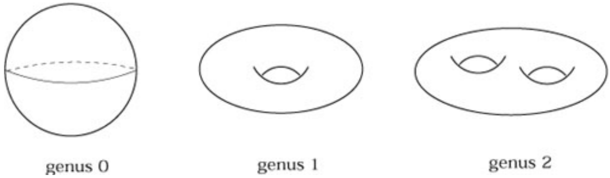

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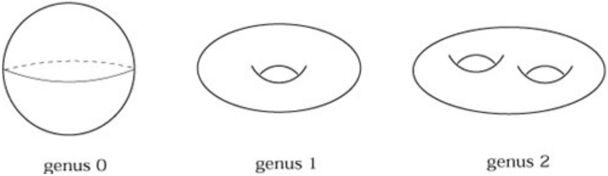

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moduli of vector bundles

Let X be an algebraic variety (e.g. $\mathbb{P}_n$, $\mathbb{P}_n \times \mathbb{P}_m, \cdots$.)

Definition

E is a vector bundle of rank r on X if it admits a surjection $p : E \to X$ such that

1. p is a linear fibration, and
2. p is a locally trivial.

- $T_{\mathbb{P}_n}$: the tangent bundle of $\mathbb{P}_n$.
- $\mathcal{O}_{\mathbb{P}_n}(-1)$: the tautological line bundle over $\mathbb{P}_n$
- $\mathcal{O}_{\mathbb{P}_n}(k)$ for $k \in \mathbb{Z}$.

Theorem (Maruyama)

There exists a moduli space $\mathcal{M}$ of stable vector bundle of rank r on X with given numeric invariants.

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Over curves

- C : a smooth projective curve of genus $g \geq 2$ over $\mathbb{C}$.
- A vector bundle E of rank r is semi-stable if

$$\frac{\deg(F)}{r'} \leq \frac{\deg(E)}{r}$$

for all sub-bundles $F \subset E$ of rank r'.
- $\mathrm{Pic}(C)$: the moduli space of line bundles on C
$\Rightarrow$ (after fixing the degree) abelian variety of dimension g.
- $SU_C(r, L)$: the moduli space of semi-stable vector bundles of rank r on C with determinant $L \in \mathrm{Pic}(C)$.

Goal : understand the geometry of $SU_C(r, L)$!!!
(King-Schofield) $SU_C(r, L)$ is rational when $(r, \deg(L)) = 1$.

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Theta maps

Let us fix L to be $\mathcal{O}_C$.

Key construction : associate to $E \in SU_C(r, \mathcal{O}_C)$ a divisor on the Jacobian

$$\Theta_E := \{L \in \mathrm{Pic}^{g-1}(C) \mid H^0(C, E \otimes L) \neq 0\}.$$

Then we have either

1. $\Theta_E = \mathrm{Pic}^{g-1}(C)$, or
2. $\Theta_E \in |r\vartheta|$, where ϑ is the canonical theta divisor on Pic^{g-1}.

It defines a rational map, called the theta map;

$$\theta : SU_C(r, \mathcal{O}_C) \dashrightarrow |r\vartheta| = \mathbb{P}_N.$$

- For $r = 2$, θ is a morphism.
- For $r \geq 4$, θ is not a morphism.
- Conjecture : For $r = 3$, θ is a morphism.

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$$\theta : SU_C(r, \mathcal{O}_C) \dashrightarrow |r\vartheta| = \mathbb{P}_N.$$

- For $r = 2$, θ is a morphism.
- For $r \geq 4$, θ is not a morphism.
- Conjecture : For $r = 3$, θ is a morphism.

Theta maps

Let us fix L to be $\mathcal{O}_C$.
Key construction : associate to $E \in SU_C(r, \mathcal{O}_C)$ a divisor on the Jacobian

$$\Theta_E := \{L \in \mathrm{Pic}^{g-1}(C) \mid H^0(C, E \otimes L) \neq 0\}.$$

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Rank 2 case

Let us consider the case $r = 2$.

- (Narasimhan-Ramanan) For $g = 2$, $\theta : SU_C(2, \mathcal{O}_C) \to |2\vartheta| = \mathbb{P}_3$ is an isomorphism.
- (Brivio-Verra, van Geemen-Izadi) For $g \geq 3$, C non-hyperelliptic, then θ is an embedding.
- (Bhosle-Ramanan) For $g \geq 3$, C hyperelliptic, then θ is a 2-to-1 finite morphism onto an explicitly described subvariety of $|2\vartheta|$.

Theorem (Narasimhan-Ramanan)

Let C be a smooth non-hyperelliptic curve of genus 3.
Via θ, $SU_C(2, \mathcal{O}_C)$ is isomorphic to a quartic hypersurface $\mathcal{Q}$ in $|2\vartheta| = \mathbb{P}_7$, singular along the Kummer variety $\mathrm{Kum}(\mathrm{Pic}^{g-1}) \Rightarrow \mathcal{Q}$ is the Coble quartic.

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Restriction

Let C be a non-hyperelliptic curve of genus 3. Then C is embedded into $\mathbb{P}_2$ as a plane quartic curve by the canonical embedding.

$$C \hookrightarrow \mathbb{P}_2 = |K_C|^*.$$

Let us fix $r = 2$ and $L = K_C$ in $SU_C(r, K_C)$.

- $\mathcal{W}^r$: the closure of the set of stable bundles E with $h^0(E) \geq r + 1$. It gives a stratification,

$$SU_C(2, K_C) \supset \mathcal{W}^0 \supset \mathcal{W}^1 \supset \cdots \supset \mathcal{W}^g.$$

Over the projective plane $\mathbb{P}_2$, we define $\mathcal{M}(k)$ to be the moduli space of stable sheaves of rank 2 on $\mathbb{P}_2$ with the Chern classes $(c_1, c_2) = (1, k)$.

- Then we can define restriction maps

$$\Phi_k : \mathcal{M}(k) \dashrightarrow \mathcal{W}^{3-k} \subset SU_C(2, K_C),$$

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Description of $\mathcal{M}(2)$

Note that $SU_C(2, K_C)$ is isomorphic to the Coble quartic $SU_C(2, \mathcal{O}_C)$.

strategy : investigate $\mathcal{M}(k)$ and the restriction maps Φ_k.

For example, let us consider the case $k = 2$.

Let S be the Veronese surface;

$$v_2 : \mathbb{P}_2 \hookrightarrow S \subset \mathbb{P}_5.$$

V : a cubic hypersurface of $\mathbb{P}_5$, which is the secant variety of $S \subset \mathbb{P}_5$.V is singular along S.

Theorem (H-)

$\mathcal{M}(2) \simeq \widetilde{V}$, where $\widetilde{V}$ is the proper transform of V in the blow up of $\mathbb{P}_5$ along S.

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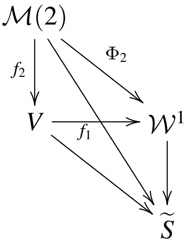

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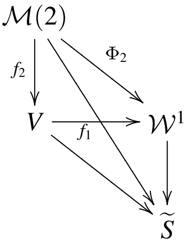

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$\Phi_4 : \mathcal{M}(4) \dashrightarrow SU_C(2, K_C)$ *is dominant.*

(Hulek) $\mathcal{M}(4)$ is a rational variety of dimension 12
$\Rightarrow$ the Coble quartic is unirational.
Let $\mathcal{H} = (H_1, \cdots, H_6)$ be a set of 6 lines on $\mathbb{P}_2$ in general position.Then we can define a sheaf $E(\mathcal{H})$ of differential 1-forms with logarithmic poles along H_i.It turns out that $E(\mathcal{H})$ is a vector bundle of rank 2 on $\mathbb{P}_2$, called the logarithmic bundle

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$\mathcal{M}(4)$ *has an open Zariski subset whose points correspond to logarithmic bundles of the above form.*

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$\Rightarrow$ the Coble quartic is unirational.
Let $\mathcal{H} = (H_1, \cdots, H_6)$ be a set of 6 lines on $\mathbb{P}_2$ in general position. Then we can define a sheaf $E(\mathcal{H})$ of differential 1-forms with logarithmic poles along H_i. It turns out that $E(\mathcal{H})$ is a vector bundle of rank 2 on $\mathbb{P}_2$, called the logarithmic bundle

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$\mathcal{M}(4)$ has an open Zariski subset whose points correspond to logarithmic bundles of the above form.

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Idea : investigate the relations between configurations of 6 lines on $\mathbb{P}_2$ and points in the Coble quartic.

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Any two general points in the Coble quartic can be connected by at most 6 rational curves of degree 10.

Right question for the conjecture

Is there a 6-dimensional rational family of 6 lines which maps to the Coble quartic birationally?

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Thank You Very Much !!!