COLLOQUIUM

Asymptotic formation and orbital stability of phase-locked states for the Kuramoto model

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Abstract

In this talk, we discuss the asymptotic formation and nonlinear orbital stability of phase-locked states arising from the ensemble of non-identical Kuramoto oscillators. We provide an explicit lower bound for a coupling strength on the formation of phase-locked states, which only depends on the diameters of natural frequencies and initial phase configurations. When the phases of non-identical oscillators are distributed over the half circle, and the coupling strength is sufficiently large, we show that the dynamics of Kuramoto oscillators exhibits two dynamic stages(initial layer and relaxation stages). In an initial layer stage, initial configurations shrink to configurations whose diameters are strictly less than ¼2 in a nite-time, and then the configurations tend to phase-locked states asymptotically. This improves previous results on the formation of phase-locked states by Chopra-Spong [1] and Ha-Ha-Kim [2] where their attention were focused only on the latter relaxation stage. We also show that the Kuramoto model is `1-contractive in the sense that the `1-distance along two smooth Kuramoto flows is less than or equal to that of initial con¯gurations. In particular, when two initial configurations have the same averaged phases, the `1-distance between them decays to zero exponentially fast. For the configurations with different phase averages, we use the method of average adjustment and translation-invariance of the Kuramoto model to show that one solution converges to the translation of the other solution exponentially fast. This establishes the orbital stability of the phase-locked states. Our stability analysis does not employ any standard linearization technique around the given phase-locked states, but we instead use a robust `1-metric functional as a Lyapunov functional. In the formation process of phase-locked states, we also estimate the number of collisions between oscillators, and lower-upper bounds of transversal phase differences.