Unique Origin
Unique Future

Mathematical Modeling Lecture

Physical Model: Projectile Motion
– an example of the mathematical modeling

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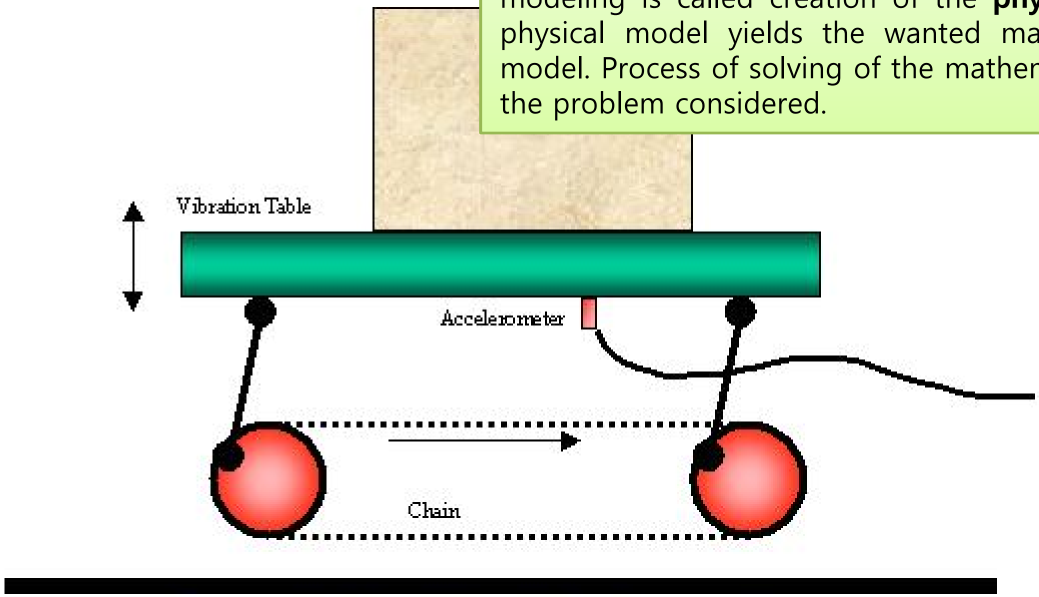
Physical Model with Mathematics

Modeling is the part of solution of an engineering problems that aims towards producing its mathematical description. This mathematical description can be obtained by taking advantage of the known laws of physics. These laws can not be directly applied to the real system. Therefore it is necessary to introduce many assumptions that simplify the engineering problems to such extend that the physic laws may be applied. This part of modeling is called creation of the **physical model**. Application of the physics law to the physical model yields the wanted mathematical description that is called mathematical model. Process of solving of the mathematical model is called analysis and yields solution to the problem considered.



Frequency and amplitude can be changed. But random is not possible.

Physical Model: Projectile Motion

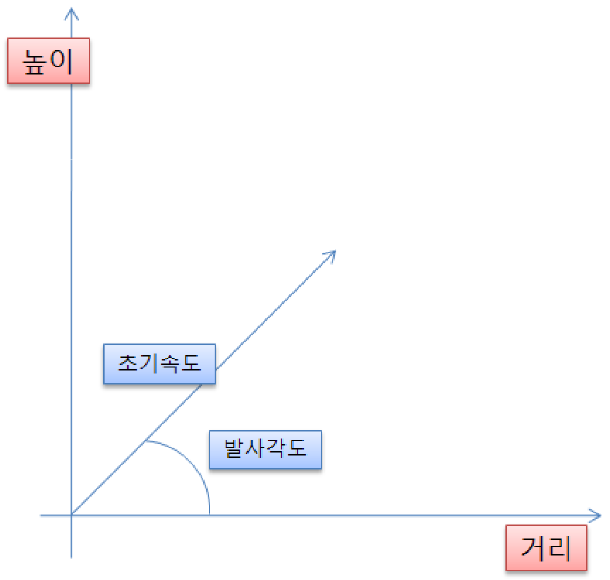
Projectile motion refers to the motion of an object projected into the air at an angle. A few examples of this include a soccer ball begin kicked, a baseball begin thrown, or an athlete long jumping. Even fireworks and water fountains are examples of projectile motion.

x 좌표의 변화량 (거리):

$$초기속도 \times \cos(발사각도)$$

y 좌표의 변화량 (높이):

$$초기속도 \times \sin(발사각도)$$



* 지구상에서는 가장 큰 물리적 힘인 중력이 작용하므로, 중력가속도만큼 높이에 영향을 미친다.

Physical Model: Projectile Motion(Example)

초기속도 30m/sec, 발사각도 40도의 Excel 모델

초기속도×cos(발사각도)

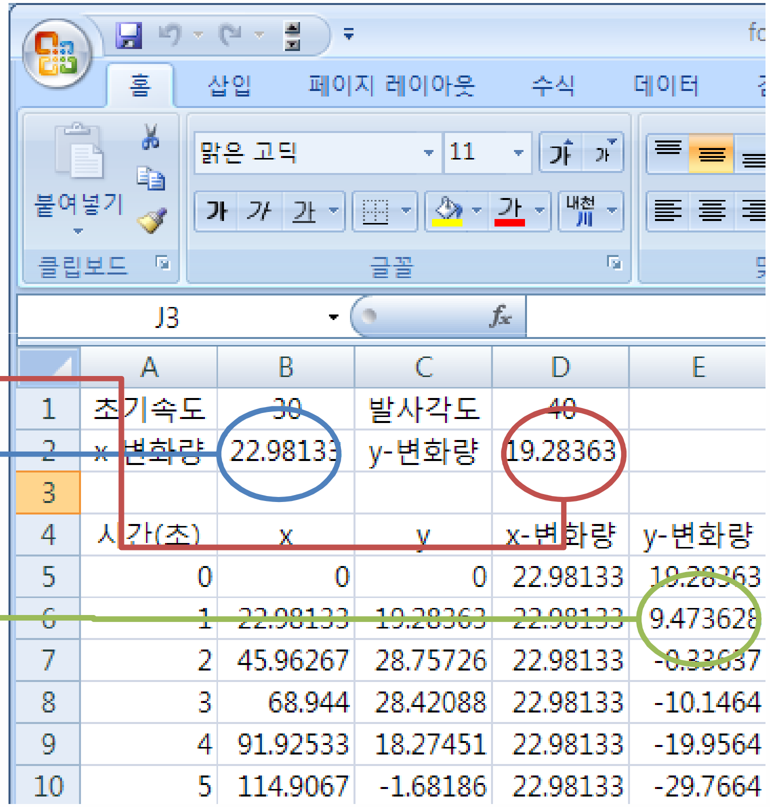
=B1*COS(RADIANS(D1))

초기속도×sin(발사각도)

=B1*SIN(RADIANS(D1))

* 지구상에서는 가장 큰 물리적 힘인 중력이 작용하므로,
중력가속도만큼 높이에 영향을 미친다.

=E5-9.81



	A	B	C	D	E
1	초기속도	30	발사각도	40	
2	x-변화량	22.98133	y-변화량	19.28363	
3					
4	시간(초)	x	y	x-변화량	y-변화량
5	0	0	0	22.98133	19.28363
6	1	22.98133	19.28363	22.98133	9.473628
7	2	45.96267	28.75726	22.98133	-0.33637
8	3	68.944	28.42088	22.98133	-10.1464
9	4	91.92533	18.27451	22.98133	-19.9564
10	5	114.9067	-1.68186	22.98133	-29.7664

Physical Model: Projectile Motion(Example)

초기속도 30m/sec, 발사각도 40도의 Excel 모델

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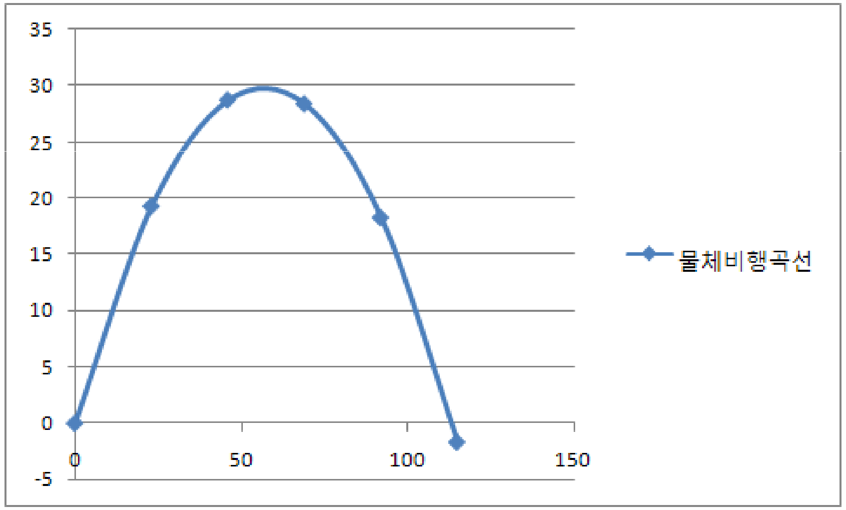
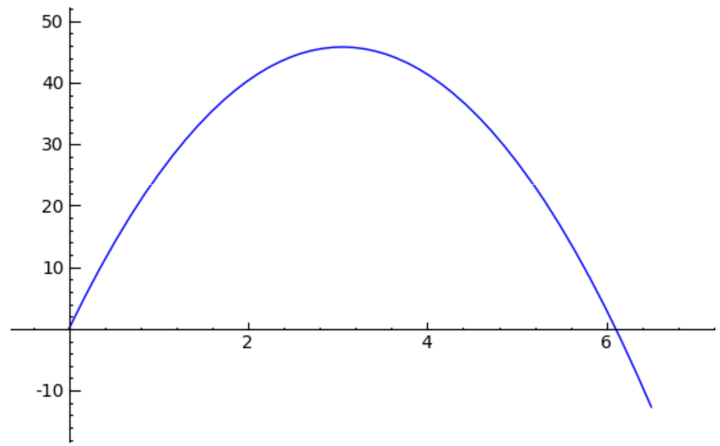


Physical Model: Projectile Motion(Mathematical Formula)

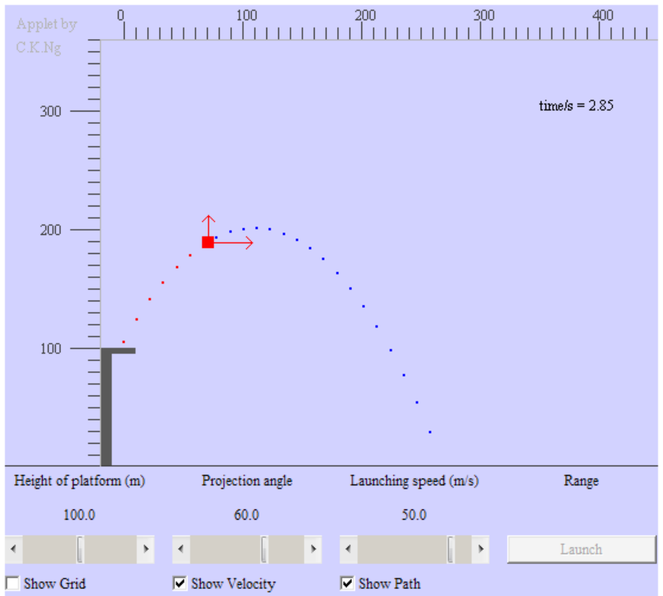
Horizontal Motion: $\dfrac{dv_x}{dt} = u\cos\theta$

Vertical Motion: $\dfrac{dv_y}{dt} = u\sin\theta - gt$

$$y = -0.5gt^2 + v_0 t + y_0$$



U : 초기속도
θ : 발사각도



Maximum Height: $H = h + \dfrac{u^2\sin^2\theta}{2g}$

Degree of Maximum Range: $\theta_{max} = \tan^{-1}\left(\dfrac{u}{\sqrt{u^2 + 2gh}}\right)$

http://www.ngsir.netfirms.com/englishhtm/ThrowABall.htm

Physical Model: Projectile Motion with Air Resistance

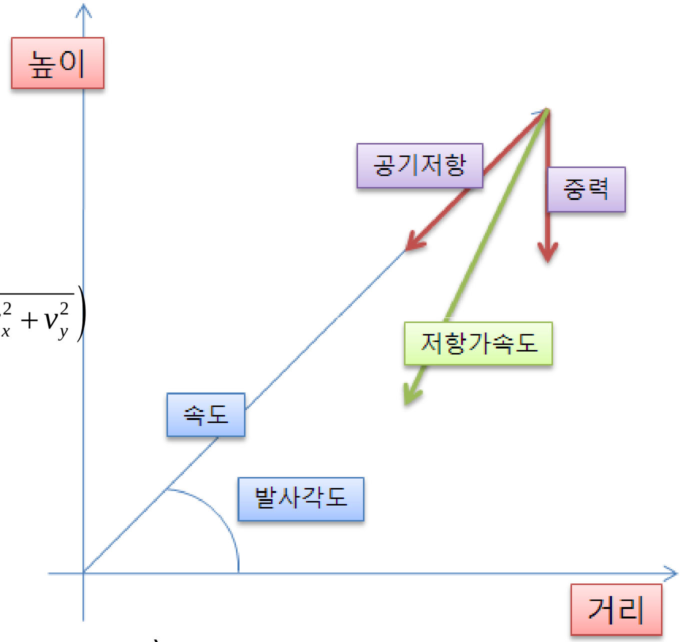
In our study of projectile motion, we assumed that air-resistance effects are negligibly small. But in fact air resistance (often called air drag, or simply drag) has a major effect on the motion of many objects, including tennis balls, bicycle riders, and airplanes.

속도벡터: $\mathbf{v} = (v_x, v_y)$

저항가속도: $\mathbf{a} = (a_x, a_y)$

중력: $\mathbf{g} = (0, -9.81)$

$$(0,-g)+\left(-cv_x\sqrt{v_x^2+v_y^2}, -v_y\sqrt{v_x^2+v_y^2}\right)$$



공기저항은 속도와 비례하므로, 상수배의 관계를 가진다.

공기저항:

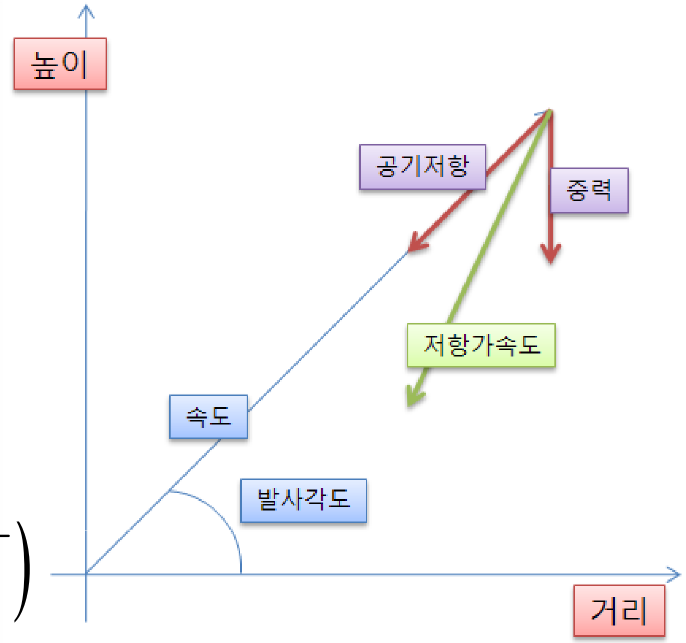
$$-c\|\mathbf{v}\|\mathbf{v} = -c\mathbf{v}\sqrt{v_x^2+v_y^2} = \left(-cv_x\sqrt{v_x^2+v_y^2}, -v_y\sqrt{v_x^2+v_y^2}\right)$$

저항가속도: 중력 + 공기저항

$$= (0,-g) + (-c\|\mathbf{v}\|\mathbf{v})$$

Physical Model: Projectile Motion with Air Resistance

How can we determine the constant c?



속도벡터: $\mathbf{v} = (v_x, v_y)$

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중력: $\mathbf{g} = (0, -9.81)$

공기저항:

$$-c\|\mathbf{v}\|\mathbf{v} = -c\mathbf{v}\sqrt{v_x^2 + v_y^2} = \left(-cv_x\sqrt{v_x^2 + v_y^2}, -cv_y\sqrt{v_x^2 + v_y^2}\right)$$

저항가속도: 중력 + 공기저항

$$= (0, -g) + (-c\|\mathbf{v}\|\mathbf{v}) = (0, -g) + \left(-cv_x\sqrt{v_x^2 + v_y^2}, -cv_y\sqrt{v_x^2 + v_y^2}\right)$$

$$= \left(-cv_x\sqrt{v_x^2 + v_y^2}, -g - cv_y\sqrt{v_x^2 + v_y^2}\right)$$

땅에 도달하면 저항가속도는 (0,0) v_x도 0이 되며 v_y가 최종속도(지면의 도달시 속도) v_{ter}의 음수가 된다.

$$(0,0) = \left(0, -g + cv_{ter}^2\right) \Rightarrow c = \frac{g}{v_{ter}^2}$$

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Physical Model: Projectile Motion with Air Resistance (Excel)

속도벡터: $\mathbf{v} = (v_x, v_y)$

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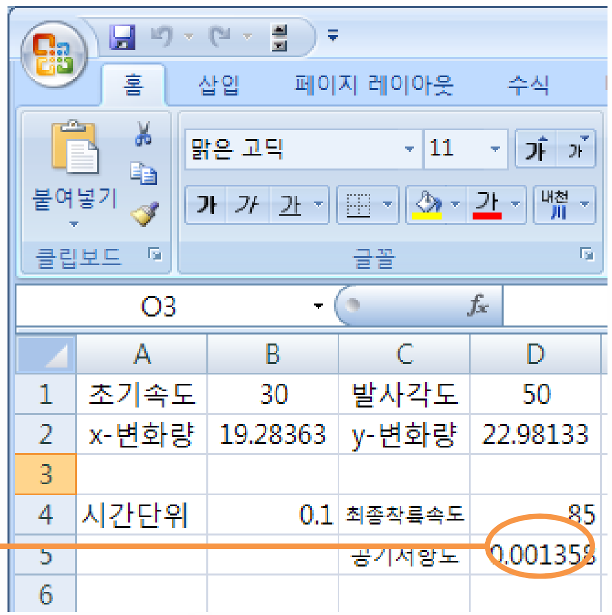
저항가속도: 중력 + 공기저항

$$= \left(-cv_x\sqrt{v_x^2 + v_y^2}, -g - cv_y\sqrt{v_x^2 + v_y^2}\right)$$

$$c = \frac{g}{v_{ter}^2}$$

=9.81/D4^2



Physical Model: Projectile Motion with Air Resistance (Excel)

3												
4	시간단위	0.1	최종착륙속도	85		지표도달속도						
5			공기저항도	0.001358								
6												
7	시간(초)	x	y	x-변화량	y-변화량	저항가속도 x변화량	저항가속도 y변화량	중력가속도 x변화량	중력가속도 y변화량	공기저항 x변화량	공기저항 y변화량	시간별속도
8	0	0	0	19.28363	22.98133	-0.78549	-10.7461	0	-9.81	-0.78549	-0.93611	30
9	0.1	1.928363	2.298133	19.20508	21.90672	-0.75969	-10.6766	0	-9.81	-0.75969	-0.86655	29.13313
10	0.2	3.848871	4.488806	19.12911	20.83907	-0.73472	-10.6104	0	-9.81	-0.73472	-0.8004	28.28762

$$c = \frac{g}{v_{ter}^2}$$

속도벡터: $\mathbf{v} = (v_x, v_y)$

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$$= \left(-cv_x\sqrt{v_x^2 + v_y^2}, -g - cv_y\sqrt{v_x^2 + v_y^2}\right)$$

Physical Model: Projectile Motion with Air Resistance (Excel with obstacle)

속도벡터: $\mathbf{v} = (v_x, v_y)$

저항가속도: $\mathbf{a} = (a_x, a_y)$

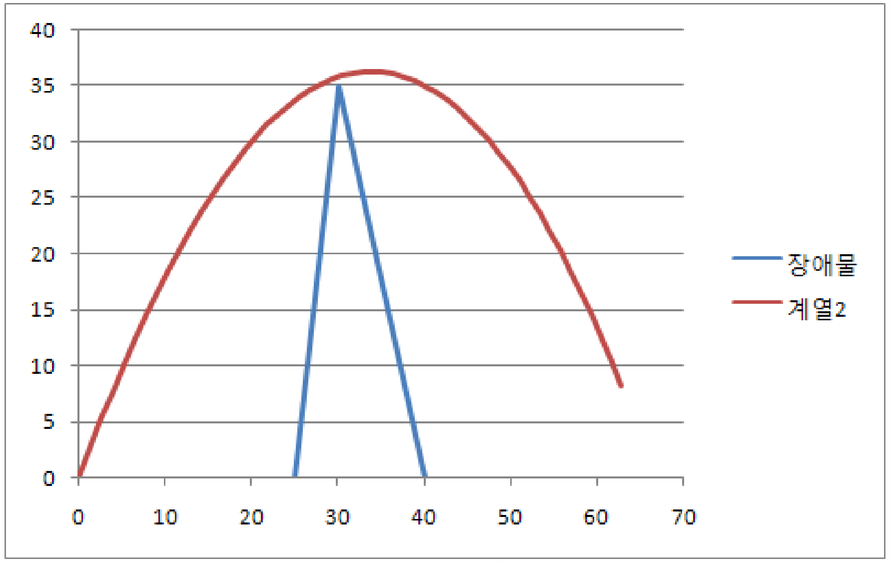
중력: $\mathbf{g} = (0, -9.81)$

공기저항:

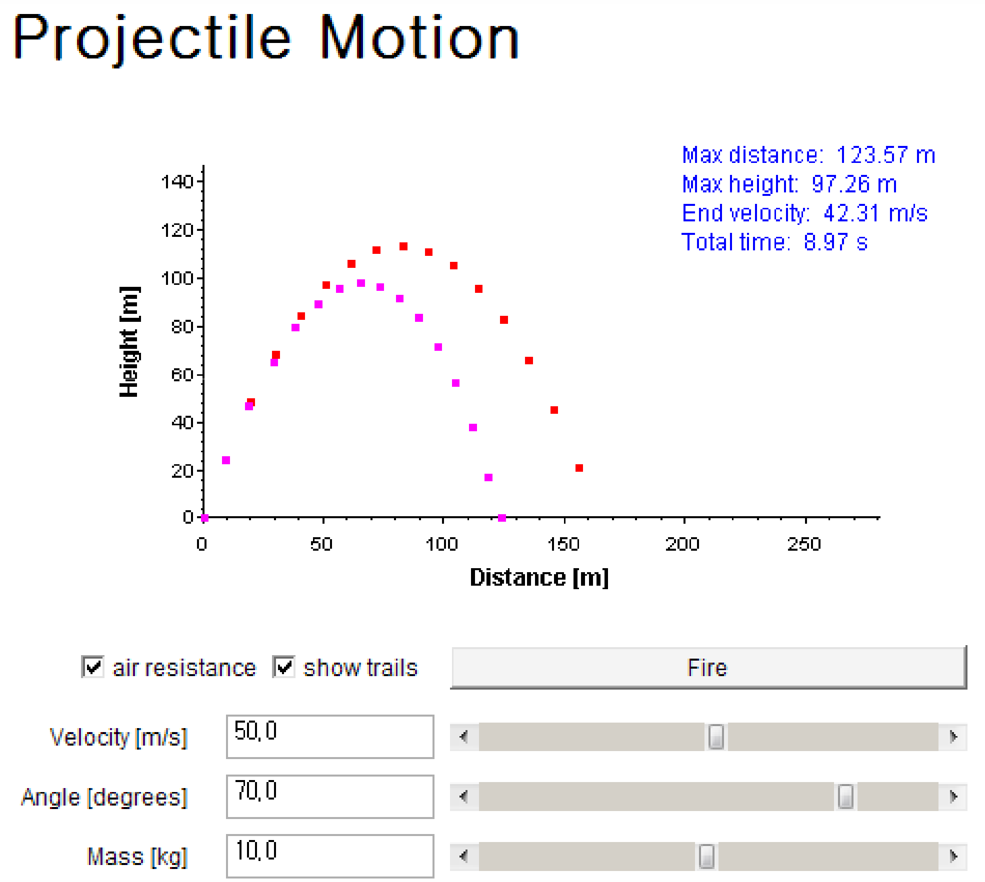
$$-c\|\mathbf{v}\|\mathbf{v} = -c\mathbf{v}\sqrt{v_x^2 + v_y^2} = \left(-cv_x\sqrt{v_x^2 + v_y^2}, -cv_y\sqrt{v_x^2 + v_y^2}\right)$$

저항가속도: 중력 + 공기저항

$$= \left(-cv_x\sqrt{v_x^2 + v_y^2}, -g - cv_y\sqrt{v_x^2 + v_y^2}\right)$$

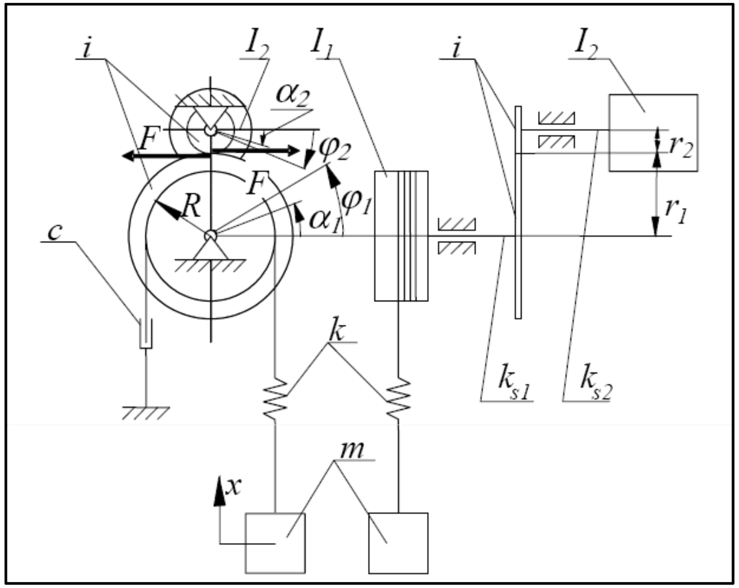


Physical Model: Projectile Motion with Air Resistance (Excel with obstacle)

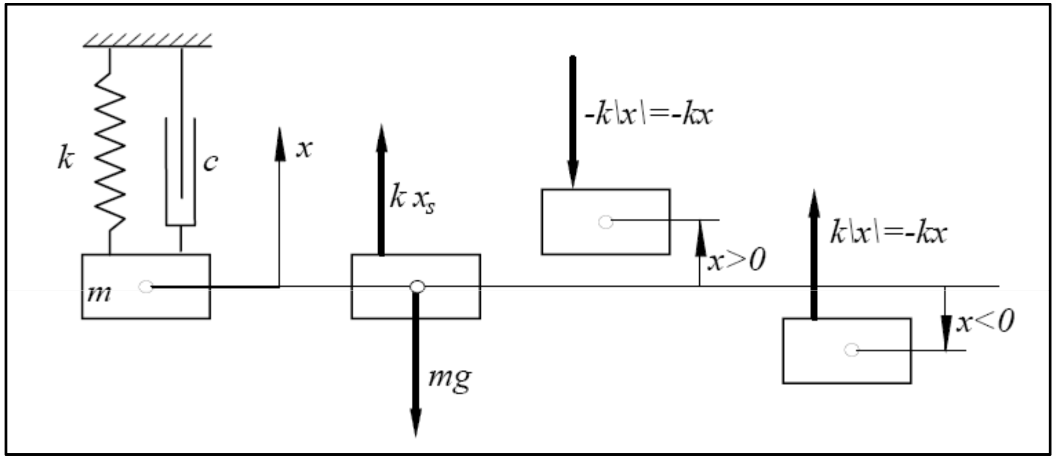


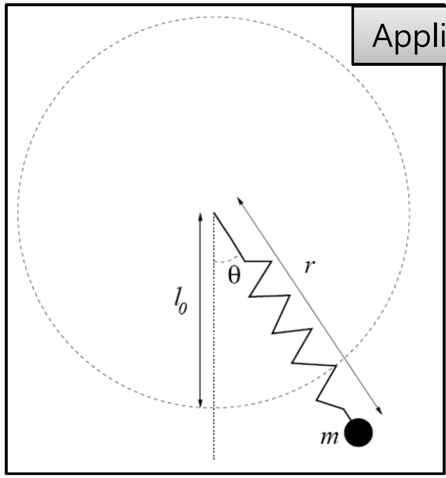
http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/ProjectileMotion/jarapplet.html

Physical Model: Applications of Projectile Motion



Complicated Mechanical System: Mechanical Vibration





Application to other mechanic system: L5 Lorentz Model(in Climate Prediction)

$$\frac{dv}{dt} = \frac{1}{pv(1+\sqrt{\epsilon}\,\rho)^2}$$

$$\frac{dp_v}{dt} = -(1+\sqrt{\epsilon}\,\rho)\sin v$$

$$\frac{d\rho}{dt} = \frac{1}{\epsilon}p_\rho$$

$$\frac{dp_\rho}{dt} = \frac{\sqrt{\epsilon}\,pv^2}{(1+\sqrt{\epsilon}\,\rho)^3} - \frac{1}{\epsilon}\rho + \sqrt{\epsilon}\cos v$$

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