1. Introduction

1.1. A bit of history ...

1.1.1. Birth of modern science and of calculus

Stage I, 1500–1630: from speculation to science ...

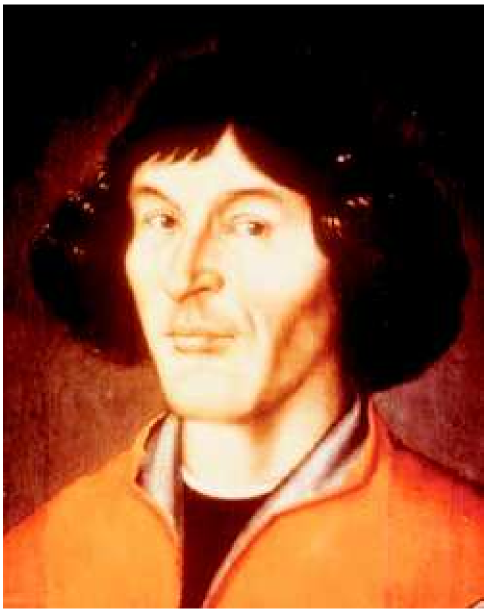
Ptolemy of Alexandria, 2nd century AD:

Style of the ancient Greeks: no experiments, just logical thought and elegance

published 'Almagest' (summary of astronomy, based on 500 years of Greek astronomical and cosmological thinking)

earth is centre of the universe
complicated model of spheres carrying heavenly bodies, moving themselves in circles

Nicolaus Copernicus, 1473–1543:



problems with the motion of the moon ...

published 'De Revolutionibus', sun-centred universe, with moon orbiting around the earth

Catholic Church:
put 'De Revolutionibus' on the Index of banned books
(stayed on the Index until 1835!)

Tycho Brahe, 1546–1601:

The genius observer...

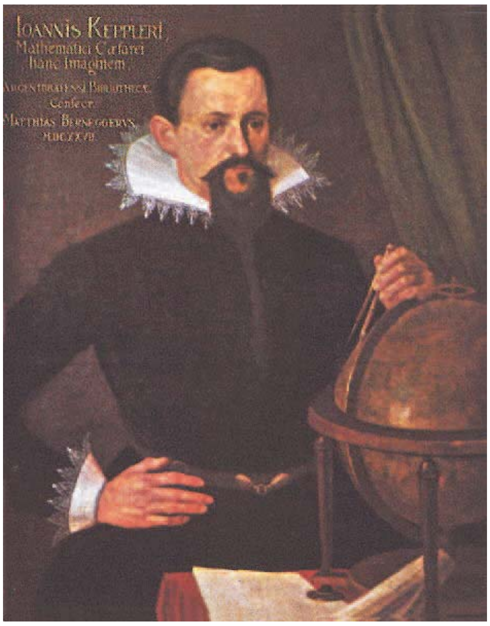
First systematic and comprehensive measurement of the trajectories of the moon, the planets, the comets, and the stars, over many years and with unrivaled precision!

Compiled *huge* amounts of data

Did *not* himself believe Copernicus' ideas ...

(lost his nose while a student in a duel in 1566)

Johannes Kepler, 1571–1630:



The genius in analyzing data ...

Believed Copernicus, but could not observe anything himself (poor eyesight ...)

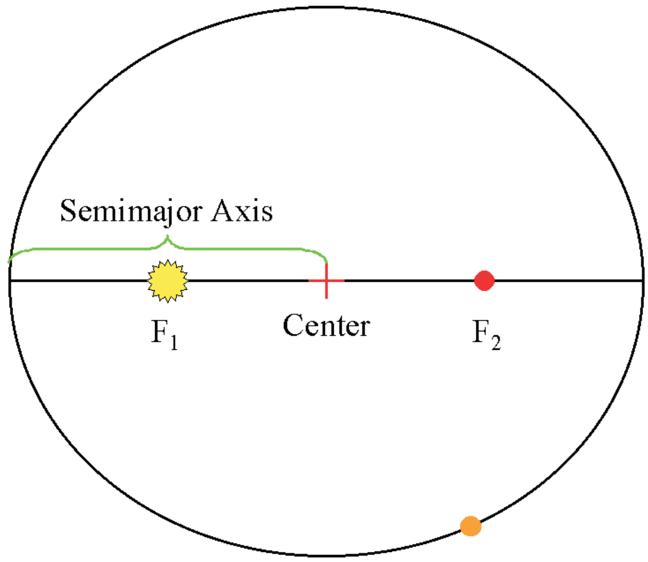
Developed further models of sun-centered universe, with spheres within spheres

Became Brahe's assistant in 1599, discovered *quantitative* laws, based on Tycho Brahe's data

published 'Astronomia Nova' in 1609, 'Harmonice Mundi' in 1619, 'Epitome of Copernican Astronomy' (3 volumes) 1618–1621

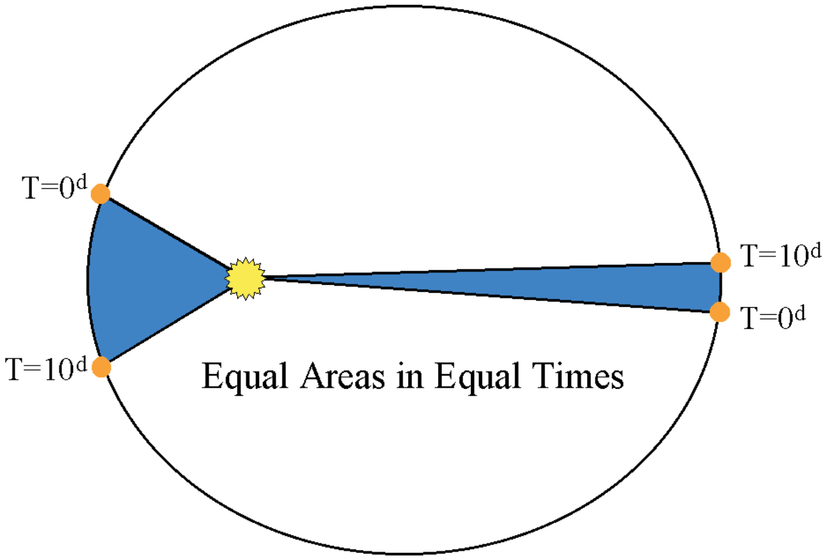
Kepler's First Law (1605):

the orbit of each planet is an ellipse, with the sun at one of the two foci



Kepler's Second Law (1602):

a line joining the sun to an orbiting planet sweeps out equal areas in equal times



Kepler's Third Law (1618):

the square of a planet's orbit period is proportional to cube of its distance to the sun

1.1.2. Birth of modern science and of calculus
Stage II, 1630–1680: science is written in the language of mathematics!

Galileo Galilei, 1564–1642:

'the wrangler', loved arguments ...

the first real scientist:
(i) state a hypothesis,
(ii) devise an experiment to test it,
(iii) carry out the experiment,
(iv) accept or reject the hypothesis

always worried about money (sisters' dowries ...)
worked on inventions to get rich
(thermometer, calculator)

interested in movement of objects
constructed improved telescope in 1609: new observations all supported Copernicus ...

published 'Dialogue on the Two Chief World Systems' in 1632
(Salviati vs Simplicio, with Sagredo as impartial commentator)

it was suggested that Pope Urban VIII was the 'simpleton' ...
1633: show trial by the Inquisition, Galileo (69 and fearing torture):
'I abjure, curse and detest my errors'

published 'Discourses and Mathematical Demonstrations Concerning Two New Sciences'
(first modern scientific textbook), smuggled out of Italy, published in 1638

René Descartes, 1596–1650:

1637: 'Discours de la Méthode pour bien conduire la raison et chercher la Vérité dans les Sciences'

invented 'Cartesian coordinates':
each position in space represented by three numbers

introduced letters x, y, z to denote
unknown quantities in mathematical problems

published 'Principia Philosophiae' (1644)

1.1.3. Birth of modern science and of calculus

Stage III, around 1680: how to speak the language of mathematics!

(i) state a hypothesis,
(ii) devise an experiment to test it,
(iii) carry out the experiment,
(iv) accept or reject the hypothesis

The problem in making it work in practice:

To test hypotheses on forces and movements of objects, one needs to be able to *calculate the trajectories* that would be caused by the assumed forces ...

1673

Christiaan Huygens: outward force on object in circular orbit of radius R is proportional to R^{-2}

1674

Robert Hooke: object that feels no force will move along a straight line (Newton's first law of motion ...)

Huygens

Hooke

Halley

Wren

1684 at the Royal Society ...

Edmond Halley, Christopher Wren, Robert Hooke

hypothesis: the sun attracts planets at distance R with a force proportional to R^{-2}

Is it possible to derive the observed motion of the planets from this inverse square law?

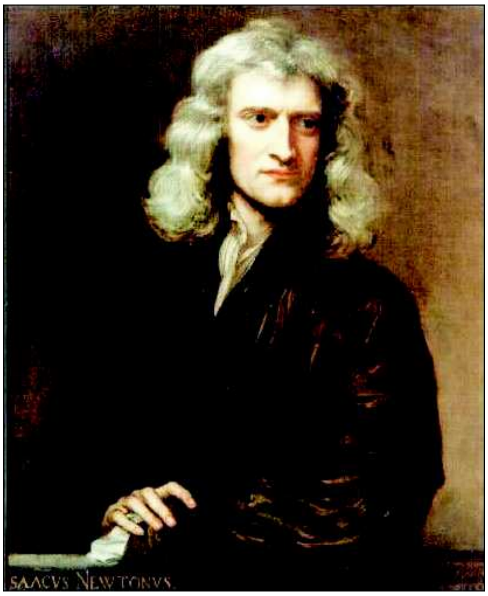
1684 somewhat later ... Halley visits Isaac Newton

according to Newton's friend De Moivre:

'Dr Halley came to visit him in Cambridge, after they had been some time together the Dr asked him what he thought the Curve would be that would be described by the Planets supposing the force of attraction towards the Sun to be reciprocal to the square of their distance from it. Sir Isaac replied immediately it would be an Ellipsis, the Dr struck with joy & amazement asked him how he knew it, why saith he, I have calculated it, whereupon Dr Halley asked him for the calculation without any further delay, Sr Isaac looked among his papers, but could not find it, but he promised him to renew it, & then send it him.'

The two parents of Calculus:

Isaac Newton , 1642–1727:



developed mechanics, calculus, theory of light
before the age of 30 ...
then spent 20 years of his life on alchemy ...

1687: publishes
'Philosophiae Naturalis Principia Mathematica'

1704: published 'Opticks'

brilliant but obsessive and nasty piece of work ...
great re-writer of history (in his own favour ...)

e.g. Hooke
(no references in Principia or Opticks!
'... by standing on the shoulders of Giants ...'
move of the Royal Society and a missing portrait)

or Leibniz
('independent commission of the Royal Society')

Gottfried Leibniz , 1646–1716:

invented calculus independently of Newton
(although slightly later)

Leibniz' notation more transparent
it is in fact what we use today!

Newton and his successors established the principles and the mode of work for *all* quantitative sciences (physics, biology, economics, etc):

- science: no longer descriptive, but aimed at finding the (usually mathematical) laws underlying the observed phenomena
- one's degree of understanding of an area of science is measured by the extent to which one can *predict* new phenomena from the discovered laws
- Galileo's principles define the procedure for finding the underlying laws. They now (i.e. after Newton and Leibniz) take the form:

  (i) state a hypothesis,
  (ii) devise an experiment to test it,
  (iii) calculate the predicted outcome of the experiment from the hypothesis
  (iv) carry out the experiment,
  (v) accept or reject the hypothesis
- When there are several distinct hypotheses, that are all consistent with the available data: select the simplest hypothesis ('Occam's Razor')

Side effects of the scientific revolution:

- industrial revolution
- 'mechanistic' view of the universe: nature is governed by differential equations
  (i) solution depends only on initial conditions
  (ii) no free will
  (iii) no divine intervention required to keep the world going ...

  of course that all changed around 1920 ...