Spring 2017, LA Midterm Exam (50 min In class Exam)							Sign
Course	Linear Algebra				Prof.	Sang-Gu Lee	
Class # (mark) 분반	41 or 42	Major 전공		Student No. 학번			Name

※ Notice : http://matrix.skku.ac.kr/LA/

1. Fill out the above boxes before you start this Exam. (학번, 이름 등을 기입하고 감독자 날인)
2. Honor Code: (시험 부정행위시 해당 교과목 성적이 "F" 처리됨은 물론 징계위원회에 회부될 수 있습니다.)
3. You can go out only after the permission from proctors. (감독위원의 지시가 있기 전에는 고사장 밖으로 나갈 수 없으며, 감독위원의 퇴실 지시가 있으면 답안지를 감독위원께 제출한 후에 퇴실하시기 바랍니다.)
4. You may use the following <Sage codes> in your answers. (중간고사까지는 한국어 답안도 OK)

Total Score (100 pt)	
Offline Exam 86	Participation 14
	14

```
var('a, b, c, d')                     # Define variables
eq1=3*a+3*b==12                       # Define equation1
eq2=5*a+2*b==13                       # Define equation2
solve([eq1, eq2], a,b)                # Solve eq's
A=matrix(QQ, 3, 3, [3, 0, 0, 0, 0, 2, 0, 3, 4]);    # Matrix
x=vector([3, 1, 2])                   # Define vector x
A.augment(x)                          #  [A: x]
A.echelon_form()  또는  A.RREF() # Find RREF
A.inverse()                           # Find  inverse
A.det()                               # Find determinant
A.adjoint()                           # Find adjoint matrix
A.charpoly()                          # Find charct. ploy
A.eigenvalues()                       # Find eigenvalues
A.eigenvectors_right()                # Find eigenvectors
A.rank()                              # Find rank of A
A.right_nullity()                     # Find nullity of A
var('t')                              # Define variables
x=2+2*t                               # Define a parametric eq.
y=-3*t-2
bool( A== B)                          # Are A and B same?

var('x, y')                           # Define variables
f = 7*x^2 + 4*x*y + 4*y^2-23          # Define a function
implicit_plot( f, (x, -10, 10), (y, -10, 10))   # implicit Plot
parametric_plot((x,y), (t, -10, 10), rgbcolor='red')  # Plot
plot3d(y^2+1-x^3-x, (x, -pi, pi), (y, -pi, pi))     # 3D Plot
A=random_matrix(QQ,7,7)   # random matrix of size 7 over Q
F=random_matrix(RDF,7,7)  # random matrix of size 7 over R
P,L,U=A.LU()              # LU  (P: Permutation M. / L, U
print P, L, U

h(x, y, z) = [x+2*y-z, y+z, x+y-2*z]
T = linear_transformation(U, U, h)   # L.T.
print T.kernel()          # Find a basis for kernel(T)
C=column_matrix([x1, x2, x3])
D=column_matrix([y1, y2, y3])
aug=D.augment(C, subdivide=True)
Q=aug.rref()

[G,mu]=A.gram_schmidt()      # G-S
B=matrix([G.row(i)/G.row(i).norm() for i in range(0,4)]); B #

A.H   # conjugate transpose of A
A.jordan_form()   # Jordan Canonical Form of A
```

<Sample Sage Linear Algebra codes>

I. (1pt x 20= 20pt) True(T) or False(F). Let S be a set of m vectors in $\mathbb{R}^n$.

1. ( ) The set of all linear combinations of two vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^n$ is a plane.

2. ( ) A set of vectors in $\mathbb{R}^n$ that contains a zero vector is linearly dependent.

3. ( ) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set, then so is the set $\{k\mathbf{v}_1, k\mathbf{v}_2, k\mathbf{v}_3\}$ for any scalar k.

4. ( ) Any $n\times n$ matrix can be written as a product of elementary matrices.

5. ( ) $T(x,y,z)=(2x+3y, -x+z-2)$ is a linear transformation.

6. ( ) If $T:\mathbb{R}^n \to \mathbb{R}^m$ is surjective, $\ker T=\{0\}$.

7. ( ) If A is invertible, 0 is an eigenvalue of A.

8. ( ) $\sigma=(3\ \ 4\ \ 1\ \ 2)$ is an odd permutation.

9. ( ) If $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ in $\mathbb{R}^3$ are linearly independent vectors, then $\det[\mathbf{v}_1 : \mathbf{v}_2 : \mathbf{v}_3]=0$.

10. ( ) If A is a $n\times n$ real orthogonal matrix, then the linear mapping $\mathbf{x}\mapsto A\mathbf{x}$ preserves length.

11. ( ) For a transformation $T:\mathbb{R}^n\to\mathbb{R}^m$, if $T(\mathbf{u})=T(\mathbf{v}) \Rightarrow \mathbf{u}=\mathbf{v}$, then it is called surjective.

12 ( ) If T is linearly independent and T is a subset of S, then S is linearly independent.

13 ( ) For any $n\times n$ matrix A with $n>1$, $\det(\operatorname{adj} A)=(\det A)^{n-1}$.

14 ( ) Let $A=[a_{ij}]\in M_{m\times n}$. Then, n columns $A^{(1)}, A^{(2)}, \cdots, A^{(n)}$ of A span a row space of A.

15 ( ) In $\mathbb{R}^n$, $m\ (>n)$ vectors are always linearly independent.

16 ( ) A line $\{\mathbf{x}_0+t\mathbf{v} \mid t\in\mathbb{R}\}$ forms a subspace through $\mathbf{x}_0$ and parallel to $\mathbf{v}$.

17 ( ) A normal vector is same as the orthogonal complement of $\mathbf{n}$, $\mathbf{n}^\perp=\{\mathbf{x}\in\mathbb{R}^n \mid \mathbf{n}\cdot\mathbf{x}=0\}$.

18. ( ) A normal vector of $z=-3x+2y+4$ is $\mathbf{n}=(-3, 2, 4)$.

19. ( ) Let A be an $n\times n$ matrix. For any i, j $(1\le i, j\le n)$ $a_{1j}C_{j1}+a_{2j}C_{j2}+\cdots+a_{nj}C_{jn}=0$. ($C_{ij}$: cofactors)

20. ( ) The homogeneous system $\sum_{j=1}^{n} a_{ij}x_j=0$ for $1\le i\le m$ always has a non trivial solution if $m>n$.

Student No.		Name	

II. (3pt x 5 = 15pt) State or Define (Fill the boxes and/or state).

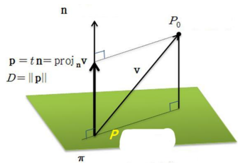
1. **Vector Equation** : A plane W in $\mathbb{R}^4$ can be uniquely obtained by passing through a point $\mathrm{x}_0 = P_0(x_0, y_0, z_0, w_0)$ and three nonzero vectors v_1, v_2 and v_3 in $\mathbb{R}^4$ that are linearly independent. Let $\mathrm{x} = P(x, y, z, w)$ be any point on W, then $\mathrm{x} - \mathrm{x}_0$ **can be expressed as a linear combination of** v_1, v_2 **and** v_3.

$\mathrm{x} - \mathrm{x}_0 =$ [   ]

where t_1, t_2 and t_3 are parameters in $\mathbb{R}$ (i.e. $-\infty < t_1, t_2, t_3 < \infty$). ■

2. For a point $P_0(x_0, y_0, z_0, w_0)$ and a plane $\pi: ax + by + cz + dw + f = 0$, **the distance** D **from the point to the plane** is

$$\mathrm{p} = \mathrm{proj}_{\mathrm{n}}\mathrm{v} = t\,\mathrm{n} = \frac{\mathrm{v} \cdot \mathrm{n}}{\mathrm{n} \cdot \mathrm{n}}\,\mathrm{n} \Rightarrow \quad D = \frac{|\qquad\qquad|}{\sqrt{a^2 + b^2 + c^2 + d^2}}$$



3. For vectors $\mathrm{x} = (x_1, x_2, \ldots, x_n)$, $\mathrm{y} = (y_1, y_2, \ldots, y_n)$ in $\mathbb{R}^n$, tell me how you can define **the angle** θ **between** x **and** y.

$|\mathrm{x} \cdot \mathrm{y}| \leq \|\mathrm{x}\|\,\|\mathrm{y}\|$ => There exist θ, $0 \leq \theta \leq \pi$ such that [   ] ■

4. [Determinant] The determinant of an $n \times n$ matrix $A = [a_{ij}]$ is defined as

[   ] ■

5. [kernel] Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then

The kernel of T = $\ker T$ =

[   ] ■

III. (3pt x 12 = 36pts) Find, Compute or Explain (Fill the boxes) :

1. Find a, b, c **of the parabolic equation** $y = ax^2 + bx + c$ which passes through $(1,3)$, $(2,5)$ and $(3,5)$. (Vandermonde matrix.)

Solution Form a LSE and use Vandermonde matrix. $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 3 \\ 5 \\ 5 \end{bmatrix}$. Then $A^{-1}\mathbf{y} = \mathbf{x}$.

A=matrix(QQ, 2, 2, [-1, 1, 2, 1]) A.det() B=A.inverse() y=matrix(QQ, 3, 1, [3, 5, 5]) By=B*y print By [-3] [-4] [-2]	A=matrix(QQ, 3, 3, [1, 1, 1, 4, 2, 1, 9, 3, 1]) A.det() B=A.inverse() y=matrix(QQ, 3, 1, [3, 5, 5]) By=B*y print By [-1] [5] [-1]

=> $|A| = -2$, $\mathbf{x} = \begin{bmatrix} -1 \\ 5 \\ -1 \end{bmatrix}$ $\therefore\ a =$ ☐ , $b =$ ☐ , $c =$ ☐ . ■

2. Find the volume of parallelepiped which is generated by three vectors, $(8,0,3)$, $(0,-4,6)$, and $(4,2,-2)$..

Solution Let $\mathbf{y}_1 = (8,0,3)$, $\mathbf{y}_2 = (0,-4,6)$, $\mathbf{y}_3 = (4,2,-2)$. The volume of parallelepiped is $\left| \det \begin{bmatrix} 8 & 0 & 3 \\ 0 & -4 & 6 \\ 4 & 2 & -2 \end{bmatrix} \right| =$ ☐ . ■

Double checked by Sage. http://math3.skku.ac.kr/home/pub/289

x1=matrix(2,1,[6,3])	a=A.det()
x2=matrix(2,1,[2,7])	b=B.det()
y1=matrix(3,1,[8,0,3])	print a.abs()
y2=matrix(3,1,[0,-4,6])	print b.abs()
y3=matrix(3,1,[4,2,-2])	
	36
A=x1.augment(x2)	
B=y1.augment(y2).augment(y3)	16

3. Find the degree 3 polynomial $y = ax^3 + bx^2 + cx + d$ which passes through the following four points.

$(0, 1)$, $(1, -1)$, $(2, -1)$, $(3, 7)$

Solution $\begin{cases} 1 = d \\ -1 = a + b + c + d \\ -1 = 8a + 4b + 2c + d \\ 7 = 27a + 9b + 3c + d \end{cases}$ => $\begin{cases} -2 = a + b + c \\ -2 = 8a + 4b + 2c \\ 6 = 27a + 9b + 3c \end{cases}$

A=matrix(3, 3, [1, 1, 1, 8, 4, 2, 27, 9, 3])	print "x=", Ai*b
b=vector([-2, -2, 6])	print "x=", A.solve_right(b)
Ai=A.inverse()	x= (1, -2, -1)

=> $a =$ ☐ , $b =$ ☐ , $c =$ ☐ , $d = 1$ $\therefore\ y =$ ☐ . ■

4. Let the characteristic polynomial of matrix A be $p(\lambda) = (\lambda - 2)(\lambda - 1)(\lambda + 3)$. **Find eigenvalues of matrix** A^3.

Solution The eigenvalues of matrix A^3 is λ^3, $\therefore$ $\lambda'_1 = 1$, $\lambda'_2 = 8$ and $\lambda'_3 =$ ☐ . ■

5. If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then **the standard matrix** $A = [T]$ **of** T has the following relation for $\mathbf{x} \in \mathbb{R}^n$. $T(\mathbf{x}) = A\mathbf{x}$, $\forall\, \mathbf{x} \in \mathbb{R}^n$ where $A =$ ☐ . ■

6. When you have eigenspaces E_1 and E_2 of $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ corresponding to each eigenvalue 0 and 5.

Show that they are orthogonal to each other in the plane.

Solution The eigenspace of A corresponding to $\lambda_1 = 0$ is $E_1 = < \begin{bmatrix} 2 \\ -1 \end{bmatrix} >$.

The eigenspace of A corresponding to $\lambda_2 = 5$ is $E_2 = < \begin{bmatrix} 1 \\ 2 \end{bmatrix} >$.

For any $\mathbf{y}_1$ in E_1 and any $\mathbf{y}_2$ in E_2, [Show $<\mathbf{y}_1, \mathbf{y}_2> = 0$]

■

7. Let A be an $n \times n$ matrix. Find **2 statements which is not equivalent to "the matrix A is invertible."? (Choose two)**

(1) Column vectors of A are linearly dependent.
(2) Row vectors of A are linearly independent.
(3) $A\mathbf{x} = \mathbf{0}$ has a unique solution $\mathbf{x} = \mathbf{0}$.
(4) For any $n \times 1$ vector $\mathbf{b}$, $A\mathbf{x} = \mathbf{b}$ has a unique solution.
(5) A and I_n are row equivalent.
(6) A and I_n are column equivalent.
(7) $\det(A) \neq 0$
(8) $\lambda = 0$ is an eigenvalue of A.
(9) $T_A : \mathbb{R}^n \to \mathbb{R}^n$ by $T_A(\mathbf{x}) = A\mathbf{x}$ is injective
(10) $T_A : \mathbb{R}^n \to \mathbb{R}^n$ by $T_A(\mathbf{x}) = A\mathbf{x}$ is surjective.

Ans ____ , ____ . ■

8. Fill out a Sage command in the box to Find eigenvalues of A and corresponding eigenvectors.

$$A = \begin{bmatrix} 4 & -1 & 0 & -1 \\ -6 & -3 & 6 & -1 \\ 0 & -2 & 4 & -2 \\ 6 & 5 & -6 & 3 \end{bmatrix}$$

```
A=matrix([[4, -1, 0, -1], [-6, -3, 6, -1], [0, -2, 4, -2], [6, 5, -6, 3]])
print ____
```

실행(Evaluate)

```
[(2, [(1, 1, 2, 1)], 1), (-2, [(0, 1, 0, -1)], 1), (4, [(1, 0, 1, 0), (0, 1, 1, -1)], 2)]
```

Answer **eigenvalues of A =**

corresponding eigenvectors :

9. Let $\mathbf{x}, \mathbf{z} \in \mathbb{R}^2$ be moved by two linear transformations T and S , where

$$T(\mathbf{x}) = \begin{bmatrix} x_1 + 2x_2 \\ x_2 \end{bmatrix} , \qquad S(\mathbf{z}) = \begin{bmatrix} z_1 \\ -z_1 + z_2 \end{bmatrix} .$$

Find $(S \circ T)(\mathbf{x})$.

Solution $[T] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $[S] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ => $[S \circ T] =$ ____ . ■

10. Find the dimension of the null space of the following matrix

$$A=\begin{bmatrix} 1 & 1 & 1 & 4 & 4 \\ 2 & 3 & 4 & 9 & 16 \\ -2 & 0 & 3 & -7 & 11 \end{bmatrix} \quad \text{where RREF}(A)=\begin{bmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -1 & 3 \end{bmatrix}.$$

Solution the dimension of the null space = [ ] ■

11. Let $[R_\theta]=\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$. Find $[R_\theta]\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$.

Solution

Answer) $[R_\theta]\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=$ [ ] ■

12. Consider $A\mathbf{x}=\mathbf{y}$ where $A=\begin{bmatrix} 7 & -2 & -2 & 1 & 0 \\ 3 & 0 & -2 & 1 & 2 \\ 12 & -4 & -3 & 2 & 0 \\ 6 & -8 & -4 & 6 & 4 \\ 1 & -2 & -2 & 1 & 6 \end{bmatrix}$ and $\mathbf{y}=\begin{bmatrix} 0 \\ 2 \\ 4 \\ 2 \\ 1 \end{bmatrix}$. You were asked to find

(1) Augment matrix [A: y] (2) RREF(A) (3) Det$A$ (4) Inverse of A (4) characteristic polynomial of A (5) all eigenvalues of A (6) all eigenvectors of A. The following is your answer. **Fill out the blanks to find each.**

1) Step 1: Browse http://math3.skku.ac.kr or http://math1.skku.ac.kr/ (or https://cloud.sagemath.com etc)
2) Step 2: Type class/your **ID: (** **) and PW : (** **)** <-- 모르면 skip if you don't have
3) Step 3: Click "New worksheet (새 워크시트)" button.
4) Step 4: Define a matrix A in the first cell in rational (QQ) field.
A = matrix(QQ,5,5,[7,-2,-2,1,0,3,0,-2,1,2,12,-4,-3,2,0,6,-8,-4,6,4,1,-2,-2,1,6]) and y = matrix(QQ, 5, 1, [0, 2, 4, 2, 1])
5) Step 5: Type a command to find augment matrix **[A: y]** A.augment(y) and evaluate
6) Step 6: Type a command to find **RREF**(A) ( ) and evaluate.
7) Step 7: Type a command to find **determinant of** A ( ) and evaluate.
8) Step 8: Type a command to find inverse of A ( ) and evaluate.
9) Step 9: Type a command to find char. polynomial of A ( ) and evaluate.
...
10) Last step : Give 'print' command to see what you like to read.

Now we have some out from the Sage.

RREF(A) = Identity matrix of size 5

det(A) = 144

inverse(A) =

```
[  -1   1/6  2/3 -1/12     0]
[ -4/3  2/3  2/3 -1/12  -1/6]`
[  -4   1/3  7/3  -1/6     0]
[ -8/3  5/6  4/3  1/12  -1/3]
[ -7/6  1/6  2/3 -1/12   1/6]
```

characteristic polynomial of (A) = x^5 - 16*x^4 + 95*x^3 - 260*x^2 + 324*x - 144

eigenvalues of A = { [ ] , [ ] , [ ] , [ ] , [ ] }

eigenvectors = [(6, [(0, 1, 0, 2, 2)], 1), (4, [(1, 1, 2, 3, 1)], 1), (3, [(1, 1, 2, 2, 1)], 1), (2, [(0, 1, 0, 2, 0)], 1), (1, [(1, 1, 3, 2, 1)], 1)]

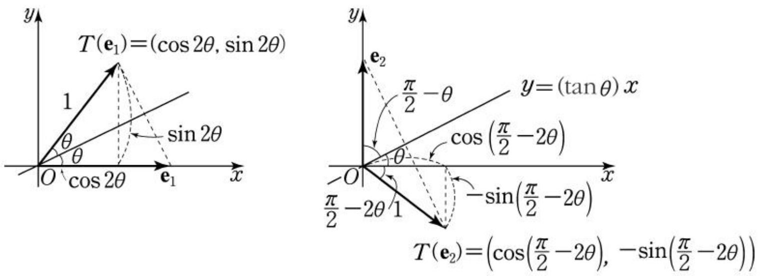
Write what (4, [(1, 1, 2, 3, 1)] , 1 **) means for Eigenvectors of** A [ ]

V. (5pt x 3 = 15pt) Explain or give a sketch of proof.

1. Define a transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ by $T(a,b,c) = \begin{bmatrix} a+b \\ 2c-a \end{bmatrix}$. Show it is a matrix transformation. (so a LT)

Solution

2. Let a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ transforms any vector $\mathbf{x} = (x, y) \in \mathbb{R}^2$ to a symmetric point to the line which passing through the origin with slope θ. Find the transformation matrix $H_\theta = [\, T(\mathbf{e}_1) \,:\, T(\mathbf{e}_2) \,]$ with the aid of following pictures.



Picture: The image of the standard basis by a symmetric transformation to the line with slope θ.

(Sol) $H_\theta = [T(\mathbf{e}_1) : T(\mathbf{e}_2)] =$ $\quad = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$. ■

3. Linear transformation (Linear operator): Let's define $T: \mathbb{R}^2 \to \mathbb{R}^2$ as a projective transformation, which transforms any vector $\mathbf{x}$ in R^2 to projection on a line which passes through the origin and has an angle θ with x-axis. For the given transformation T, let's define P_θ as a corresponding standard matrix. As shown by the right hand side picture, $P_\theta \mathbf{x} - \mathbf{x} = \frac{1}{2}(H_\theta \mathbf{x} - \mathbf{x})$ <same direction with half length>. Now by using the matrix representation of symmetric transformation $H_\theta = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$, find the standard matrix for T.

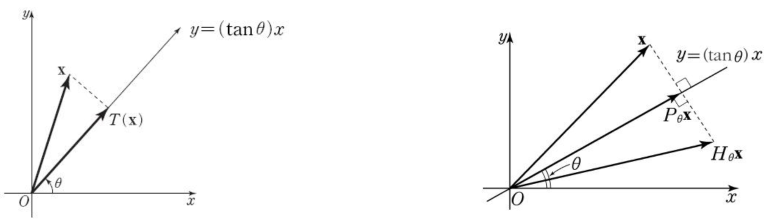


Figure The relationship between symmetric transformation and projective transformation to the line with slope θ

(Sol) $P_\theta \mathbf{x} - \mathbf{x} = \frac{1}{2}(H_\theta \mathbf{x} - \mathbf{x})$ => $P_\theta \mathbf{x} =$

=> $P_\theta = \frac{1}{2}(H_\theta + I)$ $=$ $\quad = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$ ■

Class # (mark) 분반	41 or 42

VI. LA 2017-S, Participation and more (14pt) :

Name: ____________________

<Fill this form, Print it, Bring it and submit it just before your Midterm Exam on AM (9:00, April. 19th)

(시험전에 프린트하여, 빈칸을 채워서 제출하거나, 시험 중에 확인하여 채워서 시험 시간 중에 제출하면 됩니다.)

1. (8 pt) Participations

(1) QnA Participations Numbers <Check yourself> : each weekly (From Sat – next Friday)

Week 1 : 2: 3: 4:

Week 5 : 6: 7; 8:

Total# : (Q: A:)

Online Participation : (1-8th week)

Off-line Participation/ Absence : (2*7 - 1 holiday = 13 off line classes)

(2) Your Special Contribution : (No.)

(3) What are things that you have learned and recall well from QnA and PBL participation?

2. (5 pt) Your presentation of Solutions in one Chapter.

(1) Your Team Number ( ) and Team members name ( )

(2) What was your chapter to explain to others?

(3) What was your role in that process?

(4) How you can improve your presentation for Final and Final PBL Presentation?

3. (1pt, Bonus) Write anything you like to tell.

```
var('a, b, c, d')                  # Define variables
eq1=3*a+3*b==12                    # Define equation1
eq2=5*a+2*b==13                    # Define equation2
solve([eq1, eq2], a,b)             # Solve eq's
A=matrix(QQ, 3, 3, [3, 0, 0, 0, 0, 2, 0, 3, 4]);        # Matrix
x=vector([3, 1, 2])                # Define vector x
A.augment(x)                       #   [A: x]
A.echelon_form()                   # Find RREF
A.inverse()                        # Find  inverse
A.det()                            # Find determinant
A.adjoint()                        # Find adjoint matrix
A.charpoly()                       # Find charct. ploy
A.eigenvalues()                    # Find eigenvalues
A.eigenvectors_right()             # Find eigenvectors
A.rank()                           # Find rank of A
A.right_nullity()                  # Find nullity of A
var('t')                   # Define variables
x=2+2*t                    # Define a parametric eq.
y=-3*t-2
bool( A== B)               # Are A and B same?
```

```
var('x, y')                              # Define variables
f = 7*x^2 + 4*x*y + 4*y^2-23       # Define a function
implicit_plot( f, (x, -10, 10), (y, -10, 10))   # implicit Plot
parametric_plot((x,y), (t, -10, 10), rgbcolor='red')   # Plot
plot3d(y^2+1-x^3-x, (x, -pi, pi), (y, -pi, pi))       # 3D Plot
A=random_matrix(QQ,7,7)   # random matrix of size 7 over Q
F=random_matrix(RDF,7,7)   # random matrix of size 7 over R
P,L,U=A.LU()                 # LU  (P: Permutation M. / L, U
print P, L, U

h(x, y, z) = [x+2*y-z, y+z, x+y-2*z]
T = linear_transformation(U, U, h)   # L.T.
print T.kernel()                 # Find a basis for kernel(T)
C=column_matrix([x1, x2, x3])
D=column_matrix([y1, y2, y3])
aug=D.augment(C, subdivide=True)
Q=aug.rref()

[G,mu]=A.gram_schmidt()      # G-S
B=matrix([G.row(i)/G.row(i).norm() for i in range(0,4)]); B #

A.H  # conjugate transpose of A
A.jordan_form()   # Jordan Canonical Form of A
```

<Sample Sage Linear Algebra codes>