<table>
<tr><td colspan="7">Spring 2013, Calculus I, Midterm Exam</td><td colspan="2">Sign</td></tr>
<tr><td>Course</td><td colspan="2">Calculus I with Sage</td><td>GEDB020-42</td><td>Prof.</td><td colspan="2">Sang-Gu Lee</td><td colspan="2"></td></tr>
<tr><td>Major</td><td></td><td>Year(학년)</td><td></td><td>S.N(학번)</td><td></td><td>Name</td><td colspan="2"></td></tr>
<tr><td colspan="7" rowspan="3">※ Notice (수험생 유의사항)
1. Write your name and get the signature. 답안 작성전에 이름 등을 빠짐없이 기입하고 감독자 날인을 받으세요.
2. Keep your Honor code. If not, serious penalty will be given.</td><td colspan="2">Score (150)</td></tr>
<tr><td>PBL</td><td>EXAM</td></tr>
<tr><td>/ 50</td><td></td></tr>
</table>

http://matrix.skku.ac.kr/cal-lab/sage-grapher.html , http://matrix.skku.ac.kr/cal-lab/sage-grapher-para.html

var('a,b,c,d')	# 변수정의	integral(f(x), x)	# 부정적분
limit(f(x), x=a)	# 극한	integral(f(x), x, a, b)	# 정적분
limit(f(x), x=a, dir='minus')	# 좌극한	plot(f(x), (x, a, b)	# 함수의 그래프
limit(f(x), x=a, dir='plus')	# 우극한	implicit_plot(f, (x, a, b), (y, c, d))	# 음함수 그래프
limit(f(x), x=+oo)	# 무한대에서의 극한	find_root(f(x), a, b)	# 근사해 구하기
limit(f(x), x=-oo)		var('t')	# 변수정의 (매개변수방정식)
solve(f(x)==0, x)	# Solve 방정식 풀이	x=2+2*t	
diff(f(x), x)	# 도함수	y=-3*t-2	
diff(f(x), x, 2)	# 2계 도함수	parametric_plot((x,y), (t, -10, 10), rgbcolor='red')	# 직선 Plot

I. (2pt x 12 = 24) Mark True(T) or False(F) in the blank (   ).

1. (   ) If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ also exists.

2. (   ) If $f(x)$ has a local minimum (or maximum) at $x=c$ and $f'(c)$ exists, then $f'(c)=0$.

3. (   ) Let f be continuous on $[a,b]$. Suppose $a<f(a)<b$ and $a<f(b)<b$, then there exists a number c in $(a,\ b)$ such that $f(c)=c$.

4. (   ) If $f(x)\le g(x)$ for all $a\le x\le b$, then $\int_a^b f(x)dx \le \int_a^b g(x)dx$.

5. (   ) If $\int_a^b f(x)dx = 0$, then $f(x)=0$ on $[a,\ b]$.

6. (   ) Suppose that f is continuous on the closed interval $[a,b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a)\ne f(b)$. Then there exists a number c in $(a,\ b)$ such that $f(c)=N$.

7. (   ) The most famous application of the use of hyperbolic cosine is to describe the shape of a hanging wire. If a heavy flexible cable (such as a telephone or power line) is suspended between two points at the same height, then it takes the shape of **a catenary curve** with equation $y=c+a\sinh(x/a)$, which uses the hyperbolic cosine. (The Latin word catena means **"chain."**)

8. (   ) Let f be a function whose second derivative exists on an open interval I. If $f''(x)<0$ for all x in I, then the graph of f is concave downward on I.

9. (   ) Let $C(x)$ be the cost function of producing x units of a certain product. Then the marginal cost is the rate of change of $C(x)$ with respect to x, that is, the marginal cost function is $C'(x)$. The marginal cost function is $C'(x)$ represents the cost per unit when x units are produced.

10. (   ) If $f(x)$ is not continuous on $[a,b]$, then there is no $\xi\in[a,b]$ such that $\int_a^b f(x)\,dx = f(\xi)(b-a)$.

11. (   ) If f is not a continuous function on $[a,b]$, then $F(x)=\int_a^x f(t)\,dt$ is not continuous on $[a,b]$ and $F(x)$ is not differentiable on (a,b).

12. (   ) Let f be a continuous function on $[a,b]$. Suppose that F is continuous on $[a,b]$ and that $F'=f$ on (a,b). Then $\int_a^b f(x)dx = |F(b)-F(a)|$.

II. (3pt x 5= 15) State or Define.

1. Choose only 2 terminology or concept and state its meaning as much as you knows.

Extreme Value theorem, Intermediate Value theorem, Fermat' s theorem, Mean Value theorem, Squeeze theorem, Fundamental Theorem of Calculus, [differential(미분), dy], [linear approximation(선형근사)], [Riemann Sum(리만합)], [convexity(볼록성)], [inflection point(변곡점)], [Chain Rule(연쇄법칙)], [Differentiation of Implicit function(음함수의 미분법)]

▶▶ Sol (1)

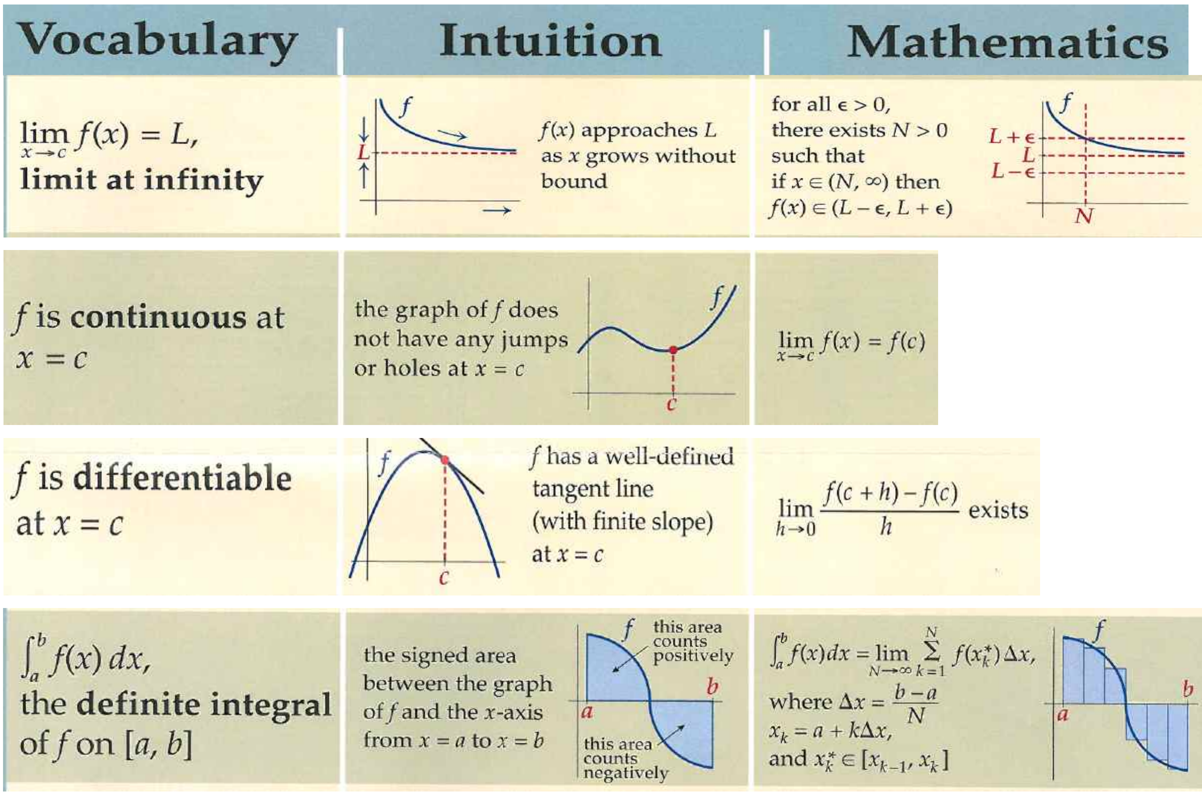
(2)

2. Discuss a geometric and physical interpretation of $f'(x)$.

▶▶ Sol Geometric interpretation :

Physical interpretation :

3. Choose 1 definition, and state in your words what it means.



	Vocabulary	Intuition	Mathematics
(1)	$\lim_{x \to c} f(x) = L$, limit at infinity	$f(x)$ approaches L as x grows without bound	for all $\epsilon > 0$, there exists $N > 0$ such that if $x \in (N, \infty)$ then $f(x) \in (L - \epsilon, L + \epsilon)$
(2)	f is continuous at $x = c$	the graph of f does not have any jumps or holes at $x = c$	$\lim_{x \to c} f(x) = f(c)$
(3)	f is differentiable at $x = c$	f has a well-defined tangent line (with finite slope) at $x = c$	$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ exists
(4)	$\int_a^b f(x)\,dx$, the definite integral of f on $[a, b]$	the signed area between the graph of f and the x-axis from $x = a$ to $x = b$	$\int_a^b f(x)dx = \lim_{N \to \infty} \sum_{k=1}^{N} f(x_k^*)\Delta x$, where $\Delta x = \frac{b-a}{N}$, $x_k = a + k\Delta x$, and $x_k^* \in [x_{k-1}, x_k]$

▶▶ Sol

4. State the Procedure for Newton's Method.

Sol Let us consider the graph of $y=f(x)$ and we want to solve $f(x)=0$. We start with the (proper, 해에 충분히 가까운) initial approximation x_1, which may be obtained by just guessing, or examining the graph of f. Then we use the tangent line L to the curve $y=f(x)$ at the point $(x_1, f(x_1))$ to approximate the curve and look at the x-intercept of L, labeled x_2. The equation of the tangent line L is

$y-f(x_1)=f'(x_1)(x-x_1)$. Thus, we obtain $0-f(x_1)=f'(x_1)(x_2-x_1)$. If $f'(x_1)\neq 0$, we can solve this equation for x_2: $x_2 = x_1 -$ ☐

Under certain conditions, x_2 is usually a better approximation to the solution than x_1. Then we repeat this procedure with x_1 replaced by x_2, using the tangent line at $(x_2, f(x_2))$. This gives a third approximation: $x_3 = x_2 -$ ☐ Continuing this process obtains a sequence of approximations x_1, x_2, x_3, ... as shown in the Figure. In general, if $f'(x_n)\neq 0$ then we have $x_{n+1}=x_n-$ ☐. The number x_n becomes closer and closer to the solution if the sequence $\{x_n\}$ converges as $n\to\infty$. We note that if $f'(x_1)\to 0$ then the sequence may not converge. In this case, we have to choose a different initial x_1.

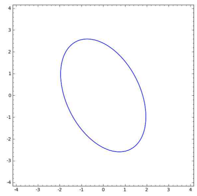
5. State what you know about the number $e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 1+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\frac{1}{1\cdot 2\cdot 3\cdot 4}+\cdots \approx 2.71828$:

Sol The number e is an important mathematical constant, approximately equal to 2.71828, that is the base of the natural logarithm. This number arises in the study of compound interest, and can also be calculated as the sum of the infinite series $e=1+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\frac{1}{1\cdot 2\cdot 3\cdot 4}+\cdots$

Ⅲ. (3pt x 13 = 39pt) Find or Explain or Fill the blank.

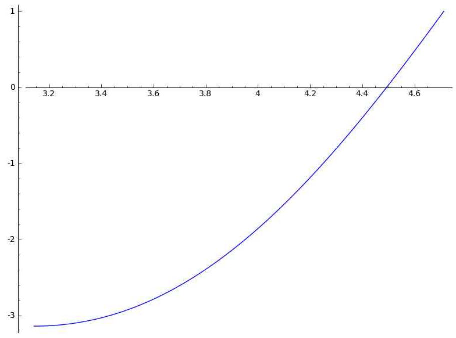
1. State the Sage command that plot the implicit function $7x^2+4xy+4y^2-23=0$ $(-4 \le x \le 4,\ -4 \le y \le 4)$.

```
var('x, y')
f = [          ]
implicit_plot(f, (x, -4, 4), (y, -4,4))
```



2. The followings explain that the equation $\sin x = x\cos x$ has at least one real root on $[\pi, 3/2\pi]$ using Intermediate Value Theorem.

```
var('x')
f(x)=(x)*cos(x) - sin(x)
plot(f(x), x, pi, (3/2)*pi)
```



```
print f(pi), f((3/2)*pi)
```

▶▶ **Sol** -pi, 1

Since f(x)=(x)*cos(x) - sin(x) is continuous on $[\pi, 3/2\pi]$ and $f(\pi) < 0 < f((3/2)*\pi)$, using Intermediate Value Theorem, there exist at least one real root of the equation $\sin x = x\cos x$ on $[\pi, 3/2\pi]$.

```
find_root(f(x),[          ]
```

▶▶ **Sol** 4.493409457909064

3. Find $f(0)$, which make $f(x)=(1+x^2)^{-\frac{5}{x^2}}$ be continuous at $x=0$. [Hint: Use $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$]

▶▶ **Sol** If $f(x)$ is continuous at $x=0$, $f(0)=\lim_{x\to 0} f(x)=\lim_{x\to 0}(1+x^2)^{-\frac{5}{x^2}}=$ [   ]

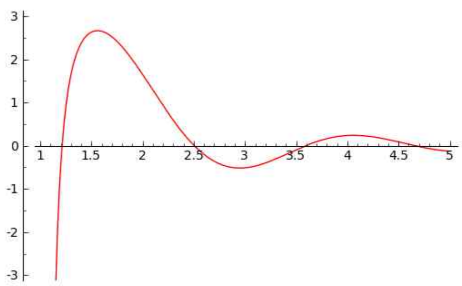
$\therefore\ f(0)=e^{-5}$.

Sage 명령어

```
x=var('x')                           # 변수 지정
limit((1+x^2)^(-5/(x^2)), x=0)
```

4. We plot the graph of the derivative of $f(x)=\dfrac{\sin 3x}{x(x-1)}$ $(1 \le x \le 5)$ using Sage below.

```
var('x')                                                  # 변수 지정
f(x)=(sin(3*x))/(x*(x-1))                                 # 함수 입력
df(x)=diff(f(x), x)                                       # 도함수 계산
plot(df(x), (x, 1, 5), ymax=3, ymin=-3, color='red')      # 도함수 그래프 그리기
```



The graph intersects the x-axis at $x=1.2,\ 2.5,\ 3.6,\ 4.7$.

(1) Find intervals on which f is decreasing.

▶▶ **Sol** $f'(x)<0$이 되는 구간에서 함수 $f(x)$가 감소하므로 위 그래프에서 $f'(x)$가 x축 아래에 위치하는 범위를 찾으면 된다.

[   ]

(2) Find x at which $f(x)$ has local extreme values

▶▶ **Sol** $f'(x)$가 존재하지 않거나 $f'(x)=0$이 되는 critical points (임계점)의 좌우에서 도함수 $f'(x)$의 값이 $(+) \to (-)$ 이면 극대, $(-) \to (+)$ 이면 극소가 된다.

[   ]

5. $\lim_{x\to 0}\frac{[f(x)-f(0)]\sin 2x}{x^2}=4$. Find $f'(0)$ [Hint: Use the definition of $f'(0)$, $\lim_{x\to 0}\frac{\sin x}{x}=1$, the properties of limits.]

Sol $4=\lim_{x\to 0}\frac{[f(x)-f(0)]\sin 2x}{x^2}=\lim_{x\to 0}\frac{f(x)-f(0)}{x}\cdot\frac{\sin 2x}{2x}\cdot 2$

$=2\left(\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}\right)\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\right)=2f'(0)\cdot 1=2f'(0)$ ($\because$ the definition of $f'(0)$)

$\therefore$ $\boxed{\qquad\qquad}$

6. Find the limit using natural logarithm and L'Hospital's Rule

$$\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$$

Sol Let $y=(\cos x)^{\frac{1}{x^2}}$ $\Rightarrow$ $\ln y=\frac{1}{x^2}\ln(\cos x)=\frac{\ln(\cos x)}{x^2}$

$\Rightarrow$ $\lim_{x\to 0}\ln y=\lim_{x\to 0}\frac{\ln(\cos x)}{x^2}=\lim_{x\to 0}\frac{-\frac{\sin x}{\cos x}}{2x}=-\frac{1}{2}\lim_{x\to 0}\frac{\tan x}{x}=-\frac{1}{2}\lim_{x\to 0}\frac{\sec^2 x}{1}$ ($\because$ L'Hospital's Rule)

$=-\frac{1}{2}\lim_{x\to 0}\sec^2 x=\boxed{\qquad}$

$\Rightarrow$ $\ln(\lim_{x\to 0}y)=\boxed{\qquad\qquad\qquad\qquad}$ ($\because$ continuous ft.)

$\therefore$ $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}=\boxed{\qquad\qquad}$

7. The tangent line at $(0,\frac{\pi}{2})$ of parametric equation $x=t\cos t$, $y=t\sin t$, $t>0$ is $y-\frac{\pi}{2}=\frac{-2}{\pi}(x-0)$ since $\left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=\frac{\pi}{2}}=-\frac{2}{\pi}$.

Find the speed (속도) and velocity (속력) at $t=\frac{\pi}{2}$.

Sol At $(0,\frac{\pi}{2})$, the speed (속도) = $\left(\frac{dx}{dt},\frac{dy}{dt}\right)=\boxed{\qquad\qquad}$ and velocity (속력) = $\sqrt{\frac{\pi^2}{4}+1}$

8. Use differential to approximate $\sqrt{98}$.

Sol Let $f(x)=\sqrt{x}$. Set $x=100$ and $\Delta x=-2$. Since $df\approx\Delta x$, $df=\frac{dx}{2\sqrt{x}}$, we have $df|_{x=100}=-\frac{1}{10}$.

Hence approximately, $\sqrt{98}=f(100+\Delta x)\approx\boxed{\qquad\qquad\qquad\qquad}=\sqrt{100}-\frac{1}{10}=\frac{99}{10}=9.9$.

9. A closed cylindrical can is to hold $500cm^3$ of liquid. Find the height and radius that minimize the amount of material needed to manufacture the can.

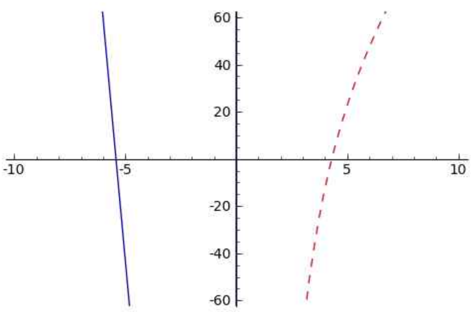
Sol $\pi r^2 h=500$ and $h=\frac{500}{\pi r^2}$

$S=2\pi r^2+2\pi rh=2\pi r^2+2\frac{500}{r}$

$\Rightarrow$ $S'=4\pi r-\frac{1000}{r^2}$

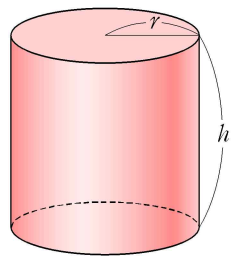
Let $S'=0$

$\Rightarrow r=\boxed{\qquad\qquad}$ and $h=\frac{500}{\pi(250/\pi)^{2/3}}=10\left(\frac{2}{\pi}\right)^{1/3}$



http://matrix.skku.ac.kr/cal-lab/cal-4-4-exs-5.html

```
var('x,y');
f=2*pi*x^2 + 1000/x
P= plot(f,x, xmin=-10, xmax=10, ymin=-60, ymax=60)
df=diff(f, x)
Q=plot(df, x, xmin=-10, xmax=10, ymin=-60, ymax=60,linestyle="--", color='red')
solve(df==0, x)    # (250/pi)^(1/3)
numerical_approx( (250/pi)^(1/3), 100) # 4.3
show(P+Q)
```



10. If $P(x)$ is the total value of the production when there are x workers in a plant, then the average productivity is $A(x) = \frac{P(x)}{x}$.

Find A'. Explain why the company wants to hire more worker if $A'(x) > 0$?

Sol $A'(x) =$

If $A'(x) > 0$, then $P'(x)x - P(x) > 0$ $(\because\ x^2 > 0)$

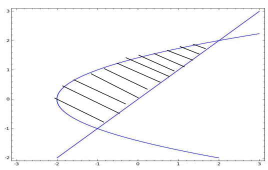
$P'(x)$ is the rate of productivity.

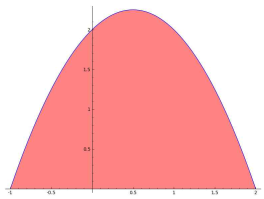
$P'(x) - \frac{P(x)}{x} > 0$ $(\because\ x > 0)$

$P'(x) > \frac{P(x)}{x}$

11. Evaluate the area covered by $y^2 = x + 2$ and $x - y = 0$.

Sol Consider $x = y$ and $x = y^2 - 2$. Area is





$\int_{-1}^{2} \{y - (y^2 - 2)\}dy$

$$\int_{-1}^{2} \{y - (y^2 - 2)\}dy = \int_{-1}^{2} (y - y^2 + 2)dy$$

$$= \left[\frac{1}{2}y^2 - \frac{1}{3}y^3 + 2y\right]_{-1}^{2}$$

$$= \frac{4}{2} - \frac{8}{3} + 4 - \frac{1}{2} - \frac{1}{3} + 2 = \frac{9}{2}$$

Sage :

```
var('y')
integral(
```

Answer : 9/2

12. $\frac{\sin x}{1 + x\sin x}$ is an anti-derivative of $f(x)$. Find $\int f(x)f'(x)dx$. [Hint: Substitute $u = f(x)$ and $du = f'(x)dx$]

$$f(x) = \left(\frac{\sin x}{1 + x\sin x}\right)' = \frac{\cos x(1 + x\sin x) - \sin x(\sin x + x\cos x)}{(1 + x\sin x)^2} = \frac{\cos x - \sin^2 x}{(1 + x\sin x)^2}$$

$$\int f(x)f'(x)dx = \int u\,du =$$

13. A honeybee population starts with 30 bees and increases at a rate of $n(t) = t^3 - 2t$ bees per week. How many honeybees are there after 10 weeks?

Sol Since the net change in population during 10 weeks is , the total number of honeybees after 10 weeks is $30 + 2400 = 2430$.

IV. (4pt x 4 = 16pt) Prove or Explain (Fill the blank).

1. $\lim_{x \to -1} x^2 + 2x - 8 = -9$

Sol $\forall\ \epsilon > 0$ [Find δ] Let $\delta =$ [   ]

If $0 < |x+1| < \delta$, then $|f(x) - (-9)| = |x^2 + 2x + 1| =$ [   ]

[Side calculation] $f(x) - f(-1) = x^2 + 2x - 8 + 9 = x^2 + 2x + 1 = (x+1)^2$. ■

2. Show $\sinh x = \dfrac{e^x - e^{-x}}{2}$, $\cosh x = \dfrac{e^x + e^{-x}}{2}$ and $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ implies $\dfrac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$.

Proof :

3. If f is a continuous function on $[a,b]$, then $F(x) = \int_a^x f(t)\,dt$ is continuous on $[a,b]$ and it is differentiable on (a,b) and $F'(x) = f(x)$.

Proof : Let x be a point in (a,b). $\dfrac{F(x+\Delta x) - F(x)}{\Delta x} = \dfrac{1}{\Delta x}\left\{\int_a^{x+\Delta x} f(t)\,dt - \int_a^x f(t)\,dt\right\}$

$= \dfrac{1}{\Delta x}\int_x^{x+\Delta x} f(t)\,dt$.

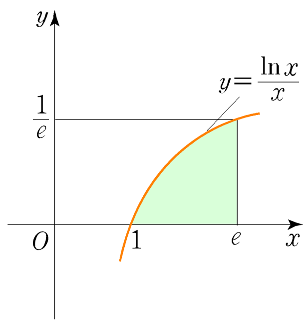
By Mean Value theorem for integration, there exist ξ in $[x, x+\Delta x]$ such that $\dfrac{1}{\Delta x}\int_x^{x+\Delta x} f(t)\,dt = f(\xi)$

Since $\xi \to x$ as $\Delta x \to 0$, and $f(x)$ is continuous. $F'(x) = \lim_{\Delta x \to 0} \dfrac{F(x+\Delta x) - F(x)}{\Delta x} = \lim_{\Delta x \to 0}$ [   ] $= f(x)$.

4. Find $\int_1^e \dfrac{\ln x}{x}\,dx$.

Sol The integrand suggests using $u = \ln x$, so then $du = dx/x$.
Now when $x = 1$, $u = \ln 1 = 0$; when $x = e$, $u = \ln e = 1$.

Thus $\int_1^e \dfrac{\ln x}{x}\,dx =$ [   ]



Figure

(QnA Participation, 4pt) **Write one good example of your Note or Solution or Answer in QnA.**

(Bonus, 2pt) **What you have newly learned and improved from our Honor Calculus with Sage?**

http://matrix.skku.ac.kr/cal-lab/sage-grapher.html , http://matrix.skku.ac.kr/cal-lab/sage-grapher-para.html

var('a,b,c,d')	# 변수정의	integral(f(x), x)	# 부정적분
limit(f(x), x=a)	# 극한	integral(f(x), x, a, b)	# 정적분
limit(f(x), x=a, dir='minus')	# 좌극한	plot(f(x), (x, a, b)	# 함수의 그래프
limit(f(x), x=a, dir='plus')	# 우극한	implicit_plot(f, (x, a, b), (y, c, d))	# 음함수 그래프
limit(f(x), x=+oo)	# 무한대에서의 극한	find_root(f(x), a, b)	# 근사해 구하기
limit(f(x), x=-oo)		var('t')	# 변수정의 (매개변수방정식)
solve(f(x)==0, x)	# Solve 방정식 풀이	x=2+2*t	
diff(f(x), x)	# 도함수	y=-3*t-2	
diff(f(x), x, 2)	# 2계 도함수	parametric_plot((x,y), (t, -10, 10), rgbcolor='red')	# 직선 Plot

Derivatives of Inverse Trigonometric Functions[1)]

$$\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x)=-\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x)=\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x)=-\frac{1}{1+x^2}$$

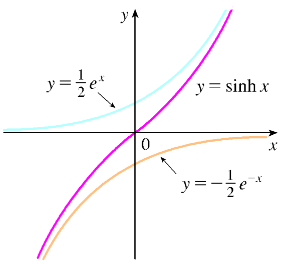
Definition of the Hyperbolic Functions[2)]

$$\sinh x=\frac{e^x-e^{-x}}{2} \qquad \cosh x=\frac{e^x+e^{-x}}{2}$$

$$\tanh x=\frac{\sinh x}{\cosh x}=\frac{e^x-e^{-x}}{e^x+e^{-x}} \qquad \coth x=\frac{\cosh x}{\sinh x}=\frac{e^x+e^{-x}}{e^x-e^{-x}}$$

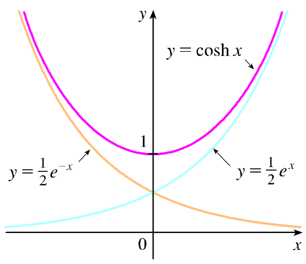
$$\operatorname{sech} x=\frac{1}{\cosh x}=\frac{2}{e^x+e^{-x}} \qquad \operatorname{csch} x=\frac{1}{\sinh x}=\frac{2}{e^x-e^{-x}}$$

The graphs of the hyperbolic sine and cosine can be sketched using graphical addition as in Figures 8 and 9.



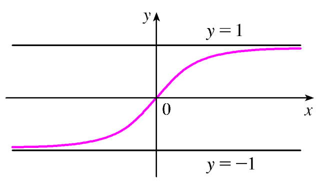
$y=\sinh x=\frac{1}{2}e^x-\frac{1}{2}e^{-x}$

Figure 8



$y=\cosh x=\frac{1}{2}e^x+\frac{1}{2}e^{-x}$

Figure 9



$y=\tanh x$

Figure 10

Derivatives of the Hyperbolic Functions

$$\frac{d}{dx}(\sinh x)=\cosh x \qquad \frac{d}{dx}(\coth x)=-\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\cosh x)=\sinh x \qquad \frac{d}{dx}(\operatorname{sech} x)=-\operatorname{sech} x\tanh x$$

$$\frac{d}{dx}(\tanh x)=\operatorname{sech}^2 x \qquad \frac{d}{dx}(\operatorname{csch} x)=-\operatorname{csch} x\coth x$$

Definition of the Inverse Hyperbolic Functions

$$\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right) \qquad (x\in R)$$

$$\cosh^{-1}x=\ln\left(x+\sqrt{x^2-1}\right) \qquad (x\ge 1)$$

$$\tanh^{-1}x=\frac{1}{2}\ln\frac{1+x}{1-x} \qquad (|x|<1)$$

$$\coth^{-1}x=\frac{1}{2}\ln\frac{x+1}{x-1} \qquad (|x|>1)$$

$$\operatorname{sech}^{-1}x=\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \qquad (0<x<1)$$

$$\operatorname{csch} x=\ln\left(\frac{1}{x}+\frac{\sqrt{1-x^2}}{|x|}\right) \qquad (x\neq 0)$$

Derivatives of the Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1}x)=\frac{1}{\sqrt{1+x^2}} \qquad (x\in R)$$

$$\frac{d}{dx}(\cosh^{-1}x)=\frac{1}{\sqrt{x^2-1}} \qquad (x>1)$$

$$\frac{d}{dx}(\tanh^{-1}x)=\frac{1}{1-x^2} \qquad (|x|<1)$$

$$\frac{d}{dx}(\coth^{-1}x)=\frac{1}{1-x^2} \qquad (|x|>1)$$

1) http://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/invtrigderivdirectory/InvTrigDeriv.html
2) http://en.wikipedia.org/wiki/Hyperbolic_function