Spring 2014, Honor Calculus I, Midterm Exam <Solution>						Sign
Course	**Honor Calculus I** with Sage	GEDB020-42	Prof.	Sang-Gu Lee		
Major		Year(학년)		S.N(학번)		Name

※ Notice (수험생 유의사항)

1. Write your name and get the signature. 답안 작성전에 이름 등을 빠짐없이 기입하고 감독자 날인을 받으세요.
2. Keep your Honor code. If not, serious penalty will be given. http://matrix.skku.ac.kr/Cal-Book/

Score (150)	
PBL	EXAM
/ 50	/100

var('a,b,c,d')	# 변수정의	solve(f(x)==0, x)	# Solve 방정식 풀이
limit(f(x), x=a)	# 극한	integral(f(x), x)	# 부정적분
limit(f(x), x=a, dir='minus')	# 좌극한	integral(f(x), x, a, b)	# 정적분
limit(f(x), x=a, dir='plus')	# 우극한	plot(f(x), (x, a, b)	# 함수의 그래프
limit(f(x), x=+oo)	# 무한대에서의 극한	implicit_plot(f(x, y), (x, a, b), (y, c, d))	# 음함수 그래프
limit(f(x), x=-oo)		find_root(f(x), a, b)	# 근사해 구하기
diff(f(x), x)	# 도함수	var('t')	# 변수정의 (매개변수방정식)
diff(f(x), x, 2)	# 2계 도함수	x=2+2*t	
bool(expr)	# 참, 거짓 판단	y=-3*t-2	
	# 여기서 expr 에는 등식 또는 부등식	parametric_plot((x,y), (t, -10, 10), rgbcolor='red')	# 직선 Plot

I. (2pt x 11 = 22) Mark True(T) or False(F) in the blank ( ).

1. (F) Given two functions f and g, the composite functions $f\circ g$, $g\circ f$ are always same. $f(x)=x+1$, $g(x)=x^2$

2. (F) The parametric equations $x=t^2$, $y=t^4$ have the same graph as $x=t^3$, $y=t^6$. The ranges of x are different.

3. (T) If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a}[f(x)g(x)]=\lim_{x\to a} f(x)\lim_{x\to a} g(x)$

4. (F) If $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ exist, then $\lim_{x\to a} f(x)$ also exists.

5. (F) If f is continuous at a, then f is differentiable at a. However, there are some functions that are differentiable but not continuous.

6. (F) Suppose $a<b$, $f(a)>0$and $f(b)<0$, then these exists a number c between a and b such that f(c)=0. $f(x)$ should be continuous.

7. (F) Air is being pumped into a spherical balloon so that its volume increases at a rate of $100\,cm^3/s$. So the radius of the balloon is increasing at the rate of $\frac{1}{20\pi}$ cm/s. $\frac{1}{25\pi}$

8. (F) Let f be a function whose first derivative exists on an open interval I. If $f'(c)=0$ where $c\in I$, then f has a local maximum or minimum at c.

9. (F) By L'Hospital's Rule, ~~$\lim_{x\to 0}\frac{x}{e^x}=\lim_{x\to 0}\frac{1}{e^x}=1$~~. $\lim_{x\to 0}\frac{x}{e^x}=\frac{\lim_{x\to 0} x}{\lim_{x\to 0} e^x}=\frac{0}{1}=1$

10. (F) If $m\le f(x)\le M$ for $a\le x\le b$, ~~$m\le\int_a^b f(x)dx\le M$~~. $m(b-a)\le\int_a^b f(x)dx\le M(b-a)$

11. (T) If $u=g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then $\int f(g(x))g'(x)dx=\int f(u)du$.

Ⅱ. (4pt x 3 = 12pt) State or Define.

1. Choose 2 terminology or concept and state its meaning as much as you knows.

Extreme Value theorem, Intermediate Value theorem, Fermat's theorem, Mean Value theorem, Squeeze theorem, Fundamental Theorem of Calculus, [differential(미분), dy], [linear approximation(선형근사)], [Riemann Sum(리만합)], [convexity(볼록성)], [inflection point(변곡점)], [Chain Rule(연쇄법칙)], [Differentiation of Implicit function(음함수의 미분법)]

▶▶ **Sol**

(1) [differential(미분), dy] Given a function $y=f(x)$ we call dy and dx **differentials** and the relationship between them is given by, $dy=f'(x)dx$. Also $f'(x)dx$ will be called **a differential of** $f(x)$ and denoted by dy or $df(x)$. Note that if we are just given $f(x)$ then the differentials are df and dx and we compute them the same manner. $df=f'(x)dx$

(2) [the Squeeze Theorem (or Sandwich Theorem)]

If $f(x)\le g(x)\le h(x)$ and $\lim_{x\to a} f(x)=\lim_{x\to a} h(x)=L$, then $\lim_{x\to a} g(x)=L$.

(3) ...

2. Choose 2 terminology or concept and state its meaning as much as you knows.

[first derivative and monotonicity(도함수와 함수의 증가, 감소)], [global maximum, global minimum(최대, 최소의 판단)], [asymtote and limit at infinity(무한대 극한과 점근선)], [Bisection method and Newton's method(이분법과 뉴튼법)], [second derivative and convexity(이계도함수와 오목, 볼록)], [critical point, local maximum, local minimum(임계점과 극대, 극소)], [equations of tangent line and normal line(접선과 법선의 방정식)]

▶▶ **Sol**

(1) Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then $\lim_{x\to a} f(x)=\infty$ means that for every positive number M there is a positive number δ such that $f(x)>M$ whenever $0<|x-a|<\delta$.

(2) A function f is continuous at $x=a$ if $\lim_{x\to a} f(x)=f(a)$. So if a function f is continuous at $x=a$, then f will satisfy the following three conditions : 1. $f(a)$ is defined (that is, a is in the domain of f) 2. $\lim_{x\to a} f(x)$ exists 3. $\lim_{x\to a} f(x)=f(a)$

(3) A function f is differentiable at a if $f'(a)$ exists.

(4) ...

State the Procedure for Newton's Method.

▶▶ **Sol** Let us consider the graph of $y=f(x)$ and we want to solve $f(x)=0$. We start with the (proper, 해에 충분히 가까운) initial approximation x_1, which may be obtained by just guessing, or examining the graph of f. Then we use the tangent line L to the curve $y=f(x)$ at the point $(x_1, f(x_1))$ to approximate the curve and look at the x-intercept of L, labeled x_2. The equation of the tangent line L is $y-f(x_1)=f'(x_1)(x-x_1)$. Thus, we obtain $0-f(x_1)=f'(x_1)(x_2-x_1)$. If $f'(x_1)\ne 0$, we can solve this equation for x_2: $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$

Under certain conditions, x_2 is usually a better approximation to the solution than x_1. Then we repeat this procedure with x_1 replaced by x_2, using the tangent line at $(x_2, f(x_2))$. This gives a third approximation: $x_3=x_2-\frac{f(x_2)}{f'(x_2)}$. Continuing this process obtains a sequence of approximations $x_1,\ x_2,\ x_3,\ \ldots$ as shown in the Figure. In general, if $f'(x_n)\ne 0$ then we have $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$. The number x_n becomes closer and closer to the solution if the sequence $\{x_n\}$ converges as $n\to\infty$. We note that if $f'(x_1)\to 0$ then the sequence may not converge. In this case, we have to choose a different initial x_1. ... ■

3. (QnA) State its meaning. Suppose that we use limit of Riemann sum to compute the Gaussian integral $\int_{-\infty}^{\infty} e^{-x^2}dx=\sqrt{\pi}$ as follows:

```
var('i, n')
f=e^(-(n^2)/2)
A=sum(f(i/n)/n, 1, n)
A.simplify
l=limit(A.simplify, n=oo)
l.expand
```

Answer: (이 문제의 경우 이런 간단한 접근으로는 답은 안나온다 ?? 그럼) 이런 경우에 어떤 코드를 써야 할까요?

보통의 적분하듯이 간단한 명령어 integrate(f, x, -a, b) 등으로 일단 수치적으로 쉽게 구할 수 있습니다.
실제 손으로 구하는 계산은 미적 2에서 배우는 중적분을 이용합니다.

```
var ('x')
f(x) = e^(-x^2)
A= numerical_integral(f, -10, 10)          # random output
B= integrate(f, x, -10, 10)                # 적분구간 숫자 10을 변형 시키면서 ...
print (A)
print (B)
```

Answer: (1.7724538509055163, 1.967819065654781e-14)

sqrt(pi)*erf(10) where erf = error function ($\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$)

이 문제가 감마분포, 정규분포, 스털링 근사, Gaussian integral 등에서 어떤 의미를 갖는지 아는대로 설명하시오.

▶▶ **Sol** icampus Q&A에서 논의 근거를 주었고, 학생들의 토론이 있었음 그것을 참고 하기 바람.

가우스가 그려진 독일 10 마르크 지폐. 가우스 얼굴 옆에 보면 가우스 적분과 정규분포 관련 식이 보인다.

가우스 적분

식 $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ 에서 평균이 0이고 표준편차가 1인 경우를 생각해 보자. 그러면 식 $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ 이 된다.

이 함수가 어떤 분포(표준 정규 분포라 부른다)의 확률 밀도 함수라는 사실은 실수 전체에서 적분한 값이 1임을 뜻한다.
이 적분을 '가우스 적분'이라고 한다.

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1 \quad 즉, \quad \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi}$$

이 가우스 적분은 조금 과장해서 통계학의 존립 기반이라 할 수 있는 '중심 극한 정리'와 관련된 적분이라는 사실만으로도 매우 중요하지만, 푸리에 이론, 편미분 방정식, 수론 등에서 약방의 감초처럼 등장하는 적분이다. 이 적분의 중요성을 알려주는 일화가 하나 더 있다. 절대온도의 단위에 이름을 남긴 영국의 물리학자 켈빈 경(William Thomson, 1st Baron Kelvin, 1824-1907)은 어느 날 수업 도중 위와 본질적으로 같은 식을 칠판에 쓰고는 이렇게 말했다고 한다.

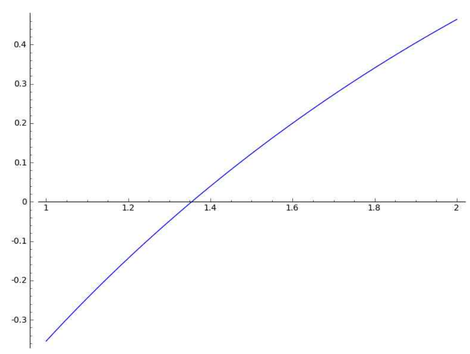
"2 곱하기 2가 4인 것이 당연하듯이, 저 식이 당연해 보이는 사람을 수학자라 부른다" ■

III. (4pt x 11 = 44pt) Find or Explain or Fill the blank.

1. The followings explain that the equation $\sqrt{8x-x^2}=3$ has **at least one real root on** $(1,2)$ **using Intermediate Value Theorem.**

```
var('x')
f(x)=sqrt(8*x - x^2) - 3
plot(f(x), x, 1, 2)
```

```
print bool( f(1)*f(2) < 0 )
```



Answer: True ■

Since $f(x)=\sqrt{8x-x^2}-3$ is continuous on $(1,2)$ and $f(1)f(2)<0$, using Intermediate Value Theorem, there exist at least one real root of the equation $\sqrt{8x-x^2}=3$ on $(1,2)$.

```
find_root(f(x), 1, 2)
```

Answer: 1.3542486889354097 ■

2. Differentiate the following function using Definition, if it exists. Then use Sage to confirm your answer.

$$g(t)=\frac{at}{bt+c}$$

Sol

$$g'(t)=\lim_{h\to 0}\frac{g(t+h)-g(t)}{h} \qquad =\lim_{h\to 0}\frac{\frac{a(h+t)}{b(h+t)+c}-\frac{at}{bt+c}}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}\frac{(ah+at)(bt+c)-at(bh+bt+c)}{(bh+bt+c)(bt+c)}=\lim_{h\to 0}\frac{1}{h}\frac{ach}{(bh+bt+c)(bt+c)}$$

$$=\lim_{h\to 0}\frac{ac}{(bh+bt+c)(bt+c)}=\frac{ac}{(bt+c)^2}$$

```
var('t, a, b, c')
f(t)=a*t/(b*t + c)
diff( f(t), t             )
```

Answer: $g'(t)=\dfrac{ac}{(bt+c)^2}$ ■

3. Is the following function differentiable at $x=0$?

$$f(x)=\begin{cases} x|x|\sin\dfrac{1}{x^2} & x\neq 0 \\ 0, & x=0\end{cases}$$

Sol The function f is differentiable at $x=0$ if and only if $f'(0)$ exists.

$$\Rightarrow\ f'(0)=\lim_{h\to 0}\frac{f(h+0)-f(0)}{h}=\lim_{h\to 0}\frac{h|h|\sin\frac{1}{h^2}}{h}=\lim_{h\to 0}|h|\sin\frac{1}{h^2}=0 \qquad \therefore \text{ The function is differentiable at } x=0$$ ■

4. Find the vertical and horizontal asymptote of the following function f and graph them together with f.

$$f(x)=\frac{x^2}{x^2-4}.$$

Sol

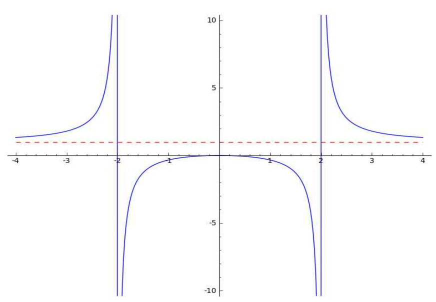
$$\lim_{x\to 2^-}f(x)=-\infty,\ \lim_{x\to 2^+}f(x)=\infty \text{ and } \lim_{x\to -2^-}f(x)=\infty,\ \lim_{x\to -2+0}f(x)=-\infty \Rightarrow \text{Vertical asymptotes : } x=\pm 2$$

$$\lim_{x\to\infty}f(x)=1 \quad \Rightarrow \text{Horizontal asymptote : } y=1.$$

[CAS] Draw the graph.

```
var('x')
f = (x^2)/(x^2-4)
p1 = plot(f, -4, 4)
p2 = plot(1, -4, 4, linestyle="--", color='red')
show(p1+p2, ymax=10, ymin=-10)
```

■



5. **Let f be a continuous function with $f(3)=0$ and $f'(3)=3$.**

Find $\lim_{x\to 0}\frac{f(3+2x)-f(3+x)}{x}$ in two ways.

▶▶ **Sol**

1) $\lim_{x\to 0}\frac{f(3+2x)-f(3+x)}{x} = \lim_{x\to 0}\frac{f(3+2x)-f(3)+f(3)-f(3+x)}{x}$

$= \lim_{x\to 0}\frac{f(3+2x)-f(3)}{x} - \frac{f(3+x)-f(3)}{x}$

$= 2\lim_{x\to 0}\frac{f(3+2x)-f(3)}{2x} - \lim_{x\to 0}\frac{f(3+x)-f(3)}{x}$

$= 2f'(3)-f'(3)=f'(3)$

2) By L'Hospital's theorem, since $\lim_{x\to 0}\{f(3+2x)-f(3+x)\}=0$ and $\lim_{x\to 0}x=0$

$\lim_{x\to 0}\frac{f(3+2x)-f(3+x)}{x} = \lim_{x\to 0}2f'(3+2x)-f'(3+x)$

$= 2f'(3)-f'(3)=f'(3)$

Answer: $\lim_{x\to 0}\frac{f(3+2x)-f(3+x)}{x}=3$ ■

6. **Use differential to approximate $\sqrt[3]{63}$.**

▶▶ **Sol** Let $f(x)=\sqrt[3]{x}$. Set $x=64$ and $\Delta x=-1$. Since $df \approx \Delta x$, $df=\frac{dx}{3\sqrt[3]{x^2}}$, we have $df|_{x=64}=-\frac{1}{48}$.

Hence approximately, $\sqrt[3]{63} = f(64+\Delta x) \approx f(64)+df|_{x=64} = \sqrt[3]{64}-\frac{1}{48}=\frac{191}{48}\approx 3.9791667$.

[CAS]

```
var('x')
f(x) = x^(1/3)
df(x) = diff(f(x), x)
dx=-1
a=f(64) + df(64)*dx
print a, "=", a.n()
```

Answer: 191/48 = 3.97916666666667 ■

7. **Find the Riemann sum by using the Midpoint Rule with the given value of n to approximate the integral.**

$$\int_1^{13}\left(\frac{5}{3}\sqrt{x}+2\right)dx,\quad n=4$$

▶▶ **Sol**

Let $f(x)=\frac{5}{3}\sqrt{x}+2$.

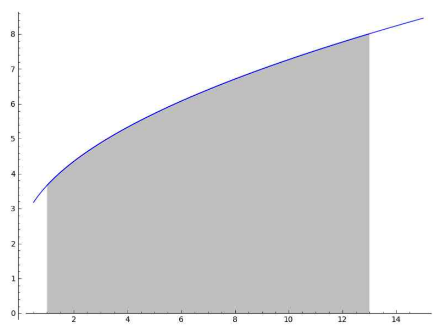
Since $n=4$, the interval width is $\triangle x=\frac{13-1}{4}=3$ and midpoints are $\overline{x_i}=3i$ for $i=1,2,3,4$.

So the Riemann sum is

$R_4=\sum_{i=1}^{4}f(x_i)\triangle x=\sum_{i=1}^{4}f(3i)3$

$=(5\sqrt{3}+6)+(5\sqrt{6}+6)+(5\sqrt{9}+6)+(5\sqrt{12}+6)$

≈ 77.22821 ■



[CAS] Draw the graph.

```
var('x')
f(x)=5/3*sqrt(x)+2
plot(f(x), (x, 1, 13), fill=true)+plot(f(x), (x, 0.5, 15))
```

8. **Find the derivative of the following function.**

$$y = \int_{3e}^{e^{nx}} \frac{\sin x^2}{e^{3x}} dx$$

Sol $y' = \dfrac{\sin(e^{nx})^2}{e^{3e^{nx}}} \cdot n e^{nx} \qquad = \dfrac{n \sin(e^{nx})^2}{e^{3e^{nx} - nx}}$ ■

9. **Find** $\lim_{x \to 0} \left(\frac{e^x + e^{-x}}{e^x - e^{-x}} - \frac{1}{x} \right)$.

Sol

$$\lim_{x \to 0} \left(\frac{e^x + e^{-x}}{e^x - e^{-x}} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{xe^x + xe^{-x} - e^x + e^{-x}}{xe^x - xe^{-x}}$$

Since $\lim_{x \to 0} (xe^x + xe^{-x} - e^x + e^{-x}) = 0$ and $\lim_{x \to 0} (xe^x - xe^{-x}) = 0$, by using L'Hospital's Rule two times we can get

$$\lim_{x \to 0} \frac{xe^x + xe^{-x} - e^x + e^{-x}}{xe^x - xe^{-x}} = \lim_{x \to 0} \frac{xe^x - xe^{-x}}{xe^x + xe^{-x} + e^x - e^{-x}}$$

$$= \lim_{x \to 0} \frac{xe^x + xe^{-x} + e^x - e^{-x}}{xe^x - xe^{-x} + 2e^x + 2e^{-x}} = 0$$

[CAS]

```
x=var('x')
f(x) = (e^(x)+e^(-x))/(e^(x)- e^(-x)) -1/x
limit(f, x=0)
```

0

Answer: $\lim_{x \to 0} \left(\frac{e^x + e^{-x}}{e^x - e^{-x}} - \frac{1}{x} \right) = 0$ ■

10. **A square has each side length** x**. Each side increases 2cm/sec. What is the rate of increase of the area of the square when the side length is 2m ?**

Sol.

A: the area of the square

$\frac{dx}{dt} = 2\,cm/\sec$

$A = x^2$ => $\frac{dA}{dt} = 2x\frac{dx}{dt} = 2 \cdot 200 \cdot 2 = 800\,cm^2/\sec$ when $x = 200cm$ ■

11. **Let** f **be a function given by** $\int x f(x) dx = \sin x + C$**. Compute** $f'(x)$**.**

Sol

$\int x f(x) dx = \sin x + C$ => $x f(x) = (\sin x + C)' = \cos x$ => $f(x) = \frac{\cos x}{x}$

=> $(x f(x))' = (\cos x)'$ => $f'(x) + \frac{f(x)}{x} = -\frac{\sin x}{x}$

=> $f'(x) = \frac{-\sin x - f(x)}{x} = -\frac{\sin x}{x} - \frac{\cos x}{x^2}$ ■

IV. (4pt x 4 = 16pt) Prove or Explain (Fill the blank).

1. $\lim_{x \to 5} \frac{1}{(x-5)^2} = \infty$.

Proof : $\forall$ **Large number** $M > 0$, **[Find δ]** **Let** $\delta = \frac{1}{\sqrt{M}}$ $(\Rightarrow \frac{1}{\delta^2} = M)$

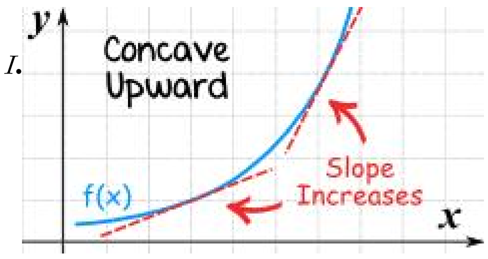
If $0 < |x-5| < \delta$, **then** $|f(x)| = \frac{1}{(x-5)^2} > \frac{1}{\delta^2} = M$

$\therefore \lim_{x \to 5} \frac{1}{(x-5)^2} = \infty$ ■

[Side calculation] Since $M > 0$, $(x-5)^2 > 0$, $\sqrt{(x-5)^2} = |x-5| > 0 \Rightarrow |x-5| < \delta$

$\frac{1}{(x-5)^2} > \frac{1}{\delta^2} = M \Rightarrow \delta = \frac{1}{\sqrt{M}}$

2. **Let f be a function whose second derivative exists on an open interval I. Show that the graph of f is concave upward (위로 오목) on I if $f''(x) > 0$ for all x in I.**



Proof : **Suppose f is not concave upward on I** when $f''(x) > 0$ for all x in I.

=> There are some $a, b \in I$ such that $a < b$ and $f'(a) \geqq f'(b)$.

By the mean value theorem, $f''(c) = \frac{f'(b) - f'(a)}{b-a}$ for some $c \in [a,b] \subset I$.

$\Rightarrow f''(c) \leq 0$ at $c \in [a,b] \subset I$. It occurs a contradiction.

$\therefore f$: concave upward on I. ■

3. **Verify the inequality**

$$\int_0^x e^t dt \geq \int_0^x (1 + t + \frac{1}{2}t^2) dt$$

Proof :

Let $f(x) = e^x - 1 - x - \frac{1}{2}xt^2$, and define $f^n(x)$ the n-th derivative function of f.

$\Rightarrow f'(x) = e^x - 1 - x \geq 0$ for $x \geq 0$

$\Rightarrow f(x) \geq 0$ ($\because f(0) = 0$)

Therefore $\int_0^x f(t) dt = \int_0^x (e^t - 1 - t - \frac{1}{2}t^2) dt \geq 0$.

$\Rightarrow \int_0^x e^t dt \geq \int_0^x (1 + t + \frac{1}{2}t^2) dt$

We have proved that $\int_0^x e^t dt \geq \int_0^x (1 + t + \frac{1}{2}t^2) dt$. ■

4. **Show a continuous and strictly decreasing function $f : \mathbb{R} \to \mathbb{R}$ has only one solution.**

Proof :

BWOC, suppose that f has more than one solution, say $a, b \in R$ such that $f(a) = f(b) = 0$ and $a < b$.

$\Rightarrow f(a) > f(b)$ ($\because f$ is strictly decreasing.) (또는 평균값정리에 의하여 $f'(c) = \frac{f'(b) - f'(a)}{b-a} = 0$)

$\Rightarrow$ It occurs a contradiction!! ($\because f(a) = 0 = f(b)$) (또는 $f'(c) > 0$ 이므로)

We have proved that a continuous and strictly decreasing function on R has only one solution. ■

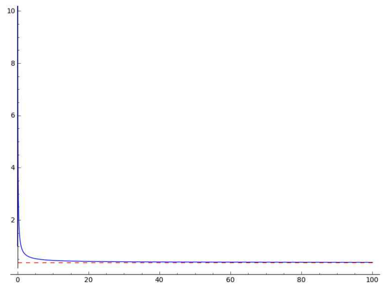
V. (2pt x 3 = 6pt) **Student Name:** ______________________ **Score: /6**

1. (2pt) Number of your QnA participation till today. (기본은 20 회 이상 - 1점 - 이고, 40회이상은 만점, 최대는 100 건)

2. (QnA, 2pt) Write one good example of your Note or Solution or Answer in QnA.

p.229 Chapter 5.6 Exercise No.7(Old)

Evaluate $\lim_{n\to\infty}(\frac{n!}{n^n})^{\frac{1}{n}}$.



Sol) $\lim_{n\to\infty}(\frac{n!}{n^n})^{\frac{1}{n}} = \lim_{n\to\infty} e^{\sum_{i=1}^{n}\frac{1}{n}\ln\frac{i}{n}} = e^{\int_0^1 \ln x dx} = e^{[x\ln x - x]_0^1} = \frac{1}{e}$

[CAS] http://math1.skku.ac.kr/home/pub/2129/

```
var('x')
f=(gamma(x+1)/(x^x))^(1/x)
P1=plot(f,x, 0, 100, ymax=10)
P2=plot(1/e, 0, 100, color='red', linestyle='--')
show(P1+P2)
```

Answer : $\lim_{n\to\infty}(\frac{n!}{n^n})^{\frac{1}{n}} = \frac{1}{e}$ ■

3. (Bonus, 2pt) What you have newly learned and improved from our Honor Calculus with Sage?

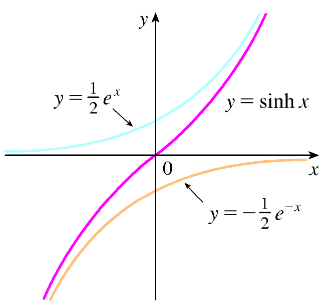
그동안 강좌를 듣고 과제를 하면서 얻은 가장 큰 것은 문제 풀이의 완성도가 늘어났다는 것이다. 과제를 몇 번 하고 PBL 보고서도 작성하며 문법적인 사소한 오류나 typing에서 오는 오류들은 거의 없어진 것 같다. 그리고 교수님께 지적받은 부분인데 영어로 답안을 쓸 때 정확히 쓰는 것이 아주 중요하다는 것을 느꼈다. 아직 수학적 영어에 익숙하지 않은 면도 있지만 필요 이상으로 장황하게 쓰는 것은 이전부터 느꼈던 것이다.

******* [별지] Honor Calculus Spring 2014, Midterm Exam *********

http://matrix.skku.ac.kr/cal-lab/sage-grapher.html , http://matrix.skku.ac.kr/cal-lab/sage-grapher-para.html

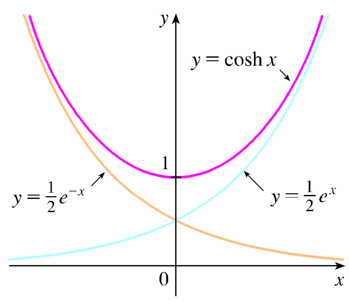
var('a,b,c,d')	# 변수정의	solve(f(x)==0, x)	# Solve 방정식 풀이
limit(f(x), x=a)	# 극한	integral(f(x), x)	# 부정적분
limit(f(x), x=a, dir='minus')	# 좌극한	integral(f(x), x, a, b)	# 정적분
limit(f(x), x=a, dir='plus')	# 우극한	plot(f(x), (x, a, b)	# 함수의 그래프
limit(f(x), x=+oo)	# 무한대에서의 극한	implicit_plot(f, (x, a, b), (y, c, d))	# 음함수 그래프
limit(f(x), x=-oo)		find_root(f(x), a, b)	# 근사해 구하기
diff(f(x), x)	# 도함수	var('t')	# 변수정의 (매개변수방정식)
diff(f(x), x, 2)	# 2계 도함수	x=2+2*t	
bool(expr)	# 참, 거짓 판단	y=-3*t-2	
	# 여기서 expr 에는 등식 또는 부등식	parametric_plot((x,y), (t, -10, 10), rgbcolor='red')	# 직선 Plot

The graphs of the hyperbolic sine and cosine can be sketched using graphical addition as in Figures 8 and 9.



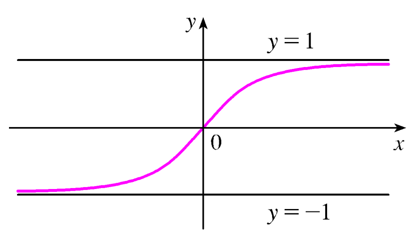
$y = \sinh x = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

Figure 8



$y = \cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

Figure 9



$y = \tanh x$

Figure 10