Chapter 4
Development and Characteristics of Korean Secondary Mathematics Textbooks

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Abstract
This study examined the ways of presenting fundamental mathematical concepts and skills in Korean secondary mathematics textbooks. Firstly, the study investigated processes of development and approval of mathematics textbooks. Secondly, it discovered eight characteristics of the secondary mathematics textbooks. Finally, in-depth interviews with five teachers were conducted to survey opinions about the new textbooks.

I. Introduction
Just as in other countries, the mathematics textbook has been the most dominant influence over what is taught in middle and high school mathematics classrooms in Korea. Classroom activities are predominantly guided, organized, and restricted to what is contained within the mathematics textbook. Thus, improving textbooks has been for many years the most important undertaking in Korean mathematics education.

Around ten kinds of mathematics textbooks are available for use in Korean secondary schools each school year. Although it was apparent that a lot of time would be needed to review the numerous textbooks and draw common features, this study was conducted during a period of 5 months, so continuation and expansion are needed after the presentation of initial findings. Korean textbooks have rapidly changed according to the curriculum reform idea. One can now find several features similar to those found in western textbooks as well as features that are uniquely different.

II. Development and Approval of Mathematics Textbook
1. Development of Textbooks
Normally university faculty members, who bring new theories, and classroom teachers, who bring empirical opinions to textbooks, participate in textbook construction. Every textbook's
quality of paper, size, way of coloring, and so on are identical as predetermined by the
government. With this fact in mind, publishing companies and textbook authors try to make
differences by incorporating a variety of approaches to mathematical concepts or operations and
editing techniques.

The main contents of Korean Secondary Mathematics Textbooks (KSMTs) are also identical
because textbook authors should conform to national curriculum and statements on school
mathematics set by the government. The national statements provide reform ideas and methods
of presenting and organizing contents with special emphasis on the following traits. First,
mathematics textbooks should consist of contents that enable students to develop mathematical
creativity, mathematical cogitation ability, and exploration ability. Second, mathematics
textbooks should include contents for improving the teaching and learning of mathematics.
Third, mathematics textbooks should be straightforward, interesting, accommodating, and
convenient for both teachers and students.

2. Approval of Textbooks

Books written for secondary school usage should be evaluated by the Ministry of
Education (MOE) to acquire utilization approval. The committee of experts who are entrusted by
MOE evaluates its quality and adequateness as a textbook. There are 20 criteria for mathematics
textbook evaluation, and they are as follows:

1. Does the textbook sufficiently reflect the nature, objectives, and contents of national
curriculum?
2. Is the textbook organized in such a manner that enables students to learn mathematics
at their own pace and thoroughly?
3. Does the book adequately select contents that allow students to attain personal learning
goals?
4. Do the level, scope, connectedness, and organization of the contents help students
learn mathematics in a stepwise and level-referenced manner?
5. Are the contents adequate for the amount of learning in one class period?
6. Does the textbook contain any mistakes or biased theories?
7. Does it present a variety of readings for students to recognize the usefulness of
mathematics?
8. Does the textbook select appropriate daily life phenomena related to mathematics and integrate other subject-matter areas into mathematics?

9. Are the following: Citizenship, humanity, environment, economy, energy, consumer, career counseling, information-oriented society, ethics, and so on properly integrated in the teaching units?

10. Does it provide effective teaching-learning methods for promoting mathematical thought and pursuit based on fundamental concepts, principles, and rules?

11. Does the textbook properly present data collection procedures, data analysis, and application as learning activities?

12. Is a variety of teaching manipulative and instructional technology for teaching-learning mathematics adequately introduced?

13. Does it present proper means of evaluation in accordance with the objectives, contents, and methods of mathematics education stated in the national curriculum?

14. Does the textbook abide by the rules of Korean spelling, the norm of standard Korean, and the orthography of foreign languages?

15. Are the mathematical terms and notations given in the national statements present and introduced accurately?

16. Does it edit and use space in an innovative way and efficiently?

17. Are the photos and illustrations appropriate and in harmony with the contents?

18. Is the facade of the textbook, including format, volume, and chromaticity in accordance with guidelines?

19. Does it use novel subject matters and seek to organize contents in an innovative way?

20. Are the processes of teaching, learning and evaluation original and fresh?

Although the above criteria could restrict textbook writers’ autonomy, it is also the means by which government-driven educational reform policy and practice are linked. In addition, it is also a powerful way of maintaining a fundamental level. On the other hand, it is very hard to comprise an appraisal committee since numerous professors and teachers participated in textbook creation. Authors who fail an initial evaluation may resubmit for government's approval, because only those approved by the appraisal committee can be published and used as textbooks in Korean secondary schools.
III. Characteristics of KSMTs

As learning elements selection is limited by the national statement, all textbooks are equivalent in scope of content, but the manner in which content is presented varies. All textbooks seem to emphasize basic concepts, principles, and rules of mathematics. Furthermore, the textbooks provide opportunity to use basic facts and skills for mathematical investigation, representation, and problem-solving. Mathematical thought and creativity are also pursued in KSMTs. All characteristics of KSMTs are shown on the following.

1. Contextual Tasks

All KSMTs seem to provide a variety of context to develop the understanding of mathematical concepts and basic skills. Real-life phenomena or contexts from other subject-matter areas have been integrated into tasks to foster the learning of fundamental mathematical concepts and skills. <Figure 1> is an example activity taken from one text that illustrates the teaching of set concept definition.

Many musical instruments produce beautiful sounds as follows.

1. Find all the string instruments.
2. Find all the wind instruments.
3. Find all the instruments producing big sound.
4. Compare the answers, if there is something that seems different, think about the reason why.

<Figure 1> A ‘definition of set’ contextual task

The definition of set is a very abstract concept, so it is hard to understand it’s meaning, yet it is the first concept students learn when they enter middle school. As a result, it plays an important role in the onset of secondary school mathematics. The activity in <Figure 1> uses the fact that most students are familiar with an array of musical instruments and the manner of classifying them. For example, most students can separate the string instruments from other
musical instruments, and consequently, they recognize that there are differences between "string instruments" and "instruments producing big sound" as a standard for classifying musical instruments. The former enables one to explicitly classify musical instruments while the latter cannot. All KSMTs writers design an assortment of context dealing with the explicitness in classification, only after a formal definition of set has been presented (Jun Yeol, Lee et al, 7-A, 8).

The next is another example of a contextual task. <Figure 2> reflects real-life phenomenon about the monthly charge method for cellular phones. Students are to fill in the given table, develop equations for finding call charges, make a graph of all the equations in the same coordinate plane, and finally find intervals which make each method of charge the most economical (Jung Ho, Woo et al, 10-B).

<table>
<thead>
<tr>
<th>Opinions</th>
<th>Basic fee (Won)</th>
<th>Fee per 10 sec (Won)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy</td>
<td>9000</td>
<td>15</td>
</tr>
<tr>
<td>Super</td>
<td>12000</td>
<td>10</td>
</tr>
<tr>
<td>Royal</td>
<td>16800</td>
<td>5</td>
</tr>
</tbody>
</table>

(1) Fill in the below table and select the most economical way for each time.

<table>
<thead>
<tr>
<th>Options</th>
<th>40min</th>
<th>80min</th>
<th>120min</th>
<th>160min</th>
<th>200min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Super</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Royal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The most economical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(2) Let represent time in minutes, and represent the call charge in 100 won. Develop equations for finding the computation of call charges. (Let represent call charge options, respectively)

(3) Make graphs of all equations on the same coordinate plane. Based on the graph, find the most economical range of time for each charge option.

<Figure 2> Contextual task related to real-life phenomena
2. Procedural Knowledge with Conceptual Background

Instruction of the principles of multiplying polynomial expressions often depends on learning by rote, and hence most KSMTs present conceptual background of the principle as in <Figure 3>(Yong Jun, Choi, 9-A).

<table>
<thead>
<tr>
<th>Learning through Communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look at the picture and answer the questions.</td>
</tr>
<tr>
<td>1. What expression is correct for each □?</td>
</tr>
<tr>
<td>((x + a)(x + b))</td>
</tr>
<tr>
<td>(= x^2 + □x + □x + ab)</td>
</tr>
<tr>
<td>(= x^2 + □x + ab)</td>
</tr>
<tr>
<td>2. What expression results when (-a) is used instead of (a), (-b) instead of (b) in the expression of Q1?</td>
</tr>
</tbody>
</table>

Students would gain conceptual and procedural knowledge about multiplication and factorization of polynomial expressions through the given geometrical representation. This approach will prevent teachers from focusing on formalization without understanding. Geometrical expression provides the conceptual basis for why the principle works. Conceptual knowledge, the understanding of the rules of multiplication and factorization of polynomial expressions, enables learners to form the connections they need to develop in algebra or complex schemas.

3. Mathematical Modeling

Mathematical modeling consists of mathematically analyzing, representing, solving and reinterpreting problem situations. It is offered in most KSMTs. Mathematical modeling is based on interconnection among mathematical topics, and the connection between mathematics and other subjects. For example, in <Figure 4>, students may be required to study environmental problems by observing number of drosophilas for 50 days, use a table and graph to represent the observed changes, then re-analyze the problem situation(KangSeob, Lee et al, Mathematics Π).
The ratio that the individual number of creatures increases in a given environment is important in environment research. The following table and graph display changes in the number of drosophilas observed over a 50 days period.

1. How many drosophilas are increased per day from the 25th to the 45th day?
2. What does the slope mean that joins point P and Q on the curve?

4. Problems Regarding Students’ Achievement Levels

KSMTs have separate problems at the end of each unit for high and low achievers. Students are to solve problems at their own level. Problems for higher level students are often complex in structure, and require students to discover advanced or original ideas in mathematics. Problems for lower level students often include simple, essential concepts or skills (YongJun, Choi, Mathematics 1). There are two groups of problems in <Figure 5>, one for slower learners and the other for faster learners. The former problems can be solved by direct application of the definition of logarithm or the laws of logarithm, while solutions to the latter problems are more complicated.
1. Simplify the following:
   (1) $\sqrt[3]{216}$  
   (2) $\sqrt[3]{2}$  
   (3) $\frac{\sqrt{2} \times \sqrt{8}}{\sqrt{16}}$

2. Select all that are correct. ($a > 0$, $a \neq 1$, $b > 0$, $c > 0$)
   I. $\log_a (b + c) = \log_a b \cdot \log_a c$
   II. $\log_a b \cdot c = \log_a b + \log_a c$
   III. $\log_a b^c = (\log_a b)^c$
   IV. $\log_a b = \frac{1}{c} \log_a b$

3. Solve the following equations for $x$ using the common logarithm table.
   (1) $\log x = 3.6749$  
   (2) $\log x = \frac{7}{2} \times 3.6749$

4. What is the value of the following equation? (Here, $[x]$ is the largest integer less than $x$)
   $[\log_3 1] + [\log_3 2] + [\log_3 3] + \cdots + [\log_3 8]$

5. $^{14}C$ (Carbon 14) has a half-life of about 5700 years. The quantity $x$ of some substance after $t$ years can be approximated by the function
   $x = x_0 \cdot 2^{-\frac{t}{h}}$, where $x_0$ is the initial value, and $h$ is the half-life of the substance. How much time must pass until the quantity of $^{14}C$ reduces to 75 g if the initial $^{14}C$ is 100 g?

5. Performance Assessment Tasks

There was strong pressure to move away from traditional multiple-choice or short-answer tests, toward alternative forms of assessment in Korea during the former curriculum period. Thus, Korean textbook writers tried to create diverse forms of tasks for assessment. The following problem in <Figure 6> assesses student ability in analysis and problem-solving when data is given(Bang Soo, Lee, 10-A).
Performance Assessment

Given data, the following task assesses the abilities of analysis and problem-solving at the same time.

Jaewon rode a bicycle to his grandmother's house, and Jaewon's elder sister, Yewon, took a bus an hour later to grandma's house since she had things to do. Yewon got to the bus stop in front of the grandmother's house 10 minutes after Jaewon had arrived. By the result of computation, the average speed Jaewon was riding his bicycle was 10km per hour, while the average speed the bus Yewon took was 40km per hour.

Activity 1. What is the distance between Jaewon’s place and his grandmother’s house?

(1) Determine the unknown variables.
(2) Develop an equation for solving the problem.
(3) Solve the equation and check if the solution is right.

Activity 2. Design a similar question and create your own solving methods.

<Figure 6> An Example of a Performance Assessment Task

6. Additional Resource Materials

Additional resource materials related to crucial concepts and skills are presented in all KSMTs. Advanced level contents or explanations corresponding to the ones in the textbooks(see <Figure 7>), anecdotes of famous Mathematicians(see <Figure 8>), historical development of mathematical topics, and so on are used to foster students' mathematics learning.
The cycloid is the locus of a point on the rim of a circle \( x^2 + y^2 = 1 \) rolling along a horizontal line without slipping.

Let the coordinates of any point \( P \) on the circumference of the circle be \( (x, y) \) in the above diagram, where \( t \) is the angle that the circle moved through. Then we have,

\[
x = t - \sin t, \quad y = 1 - \cos t
\]

which is the curve described in question 2.

There are also several other curves like astroids, and cardioids which are interesting.

Who is he?

The founder of Greek geometry, Thales discovered that the circumferential angle to the diameter of a circle is a right angle.

Information

- Relation between a circumferential angle and an arc
- The relation between a circumferential angle and an arc can be found directly.
7. Using Technology

The Korean national statement encourages the use of technology in teaching and learning in a variety of ways. Research shows that the use of technology in mathematics teaching causes students to become better problem solvers. KSMTs provide problems which can be solved by calculators or computers, and readings about alternative methods of solving problems by using technology. There are no independent sections on technology usage in KSMTs, except for the "Real-Life Mathematics Textbook," which is used by 12th grade students who selected the subject. One example using technology is shown on the following page.

Planets Named After Koreans

Minor planets discovered by Korean astronomers will be named after Koreans for the first time.

The Ministry of Science and Technology supposed to designate the minor planet 23880 discovered by a Korean amateur astronomer Tongil, which means unification, to IAU(International Astronomical Union). The discoverer of an asteroid is invited by the SBNC, a Committee on Small Body Nomenclature, to suggest a name for the asteroid. The suggested name would be approved in 2 months if there were no associated problems. The diameter of asteroid 23880 discovered by Tongil is about 5~10 km, and was located about 250,000,000 km away from the Earth when discovered. Nowadays, it is traveling around the sun at a distance away of 350,000,000 km, and its magnitude is 17.4.

Asteroids are small planets traveling around the sun with its own orbit and orbital period. 30,000 asteroids have been discovered, and the asteroid discovered by Tongil lies in the Virgin between Mars and Jupiter. It has an orbital period

<Figure 9> Additional Resource on Asteroids
Let's make a correlation table using computer

A correlation table can be made as follows:

1. Make a table
2. Select "Pivot table report"

3. Select [Next] in the first step of "Pivot table wizard" and select [Next] in the second step if the range is correct.
4. Draw "dog" to the "row," "apartment" to the "column," and "frequency" to "data," then select [Finish].

5. Result

<table>
<thead>
<tr>
<th>Apartment</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dog</td>
<td>3</td>
</tr>
<tr>
<td>Cat</td>
<td>2</td>
</tr>
<tr>
<td>Fish</td>
<td>1</td>
</tr>
</tbody>
</table>

<Figure 10> Textbook Usage of Technology

8. Connection and Repetition

Secondary school mathematics builds on the concepts, skills and understanding developed in lower grades. Hence every chapter of KSMTs begins with a diagnosis of student understanding of previously learned concepts and skills. It aims to diagnose learner readiness and provide opportunities to prepare for learning of new contents by linking previous knowledge with current knowledge. Students are provided many chances to connect new concepts and procedures to their existing knowledge base while they learn from the textbook. For example, problems related to proportional expression, corresponding table, and graph are given before students begin learning linear function(Gyu Hong, Park et al, 7-A).
Repetition is an essential way that KSMTs are organized. Similar questions are repeatedly given in a chapter. Repetition aims to develop speed and accuracy in computation. However, since over-repetition is often regarded as a negative way of teaching and learning, drill exercises are rarely found in KSMTs.

<Connection to prior study>
1. Find the value of $x$ in the next proportions.
   
   (1) $x : 3 = 6 : 9$
   
   (2) $8 : 5 = x : 10$

2. Fill in the table.

3. Draw a mathematical relationship based on the next table for $x, y$.

4. The table shows the monthly average temperature of the west coast of the country for January to December. Draw a line graph of the data.

<Figure 11> Connection to Prior Knowledge

IV. Teachers’ Guidebooks

Every textbook that has been approved for use by the Ministry of Education must develop teachers’ guidebooks and submit it for approval. Korean teacher guides usually provide explanations on curriculum reform intentions and the reasons for topics selection and sequence, specific information about the nature of topics and student responses, theories of teaching and learning mathematics, additional resource or teaching materials including higher level mathematics, historical background, alternative methods of teaching or problem-solving, means and problem sets for assessment, lesson plans for each unit, noteworthy things while teaching specific topics, efficient approaches to instruction, solutions, and so on.
Let's take a look at one Korean teachers’ guidebook for grade 12 (Jung Ho, Woo et al). There are two parts, one is general and the other is concrete:

[Contents of the General Section]
1. Features of development in mathematics and modern mathematics
2. Theories of mathematical teaching and learning
3. Assessment in mathematics education
4. Use of computers in mathematical teaching and learning
5. Reform movements in mathematics education
6. Historical overview of Korean school mathematics curricula
7. Background of the current (7th) school mathematics curriculum
8. Overview of the textbook and teacher’s guide
9. Yearly lesson plans
10. Daily lesson plan example
11. References

[Contents of the Concrete Section]
1. Overview of the unit
   (1) Goal of the unit
   (2) Connection to other units
2. Theoretical background of the unit
3. Lesson plan for the unit
4. Mathematics in the unit
   (1) Main objectives
   (2) Teaching notes
   (3) Solutions and section focus
   (4) Additional resources
5. Assessment tasks
V. Teachers’ Opinions on KSMTs

Five teachers were asked to participate in an in-depth interview to explore individual experiences related to KSMTs. The interviews were conducted over a two-week period, open-ended in format, and general in framework. The teachers provided the following suggestions.

First, although level differentiated tasks are provided in the textbooks, but students do not normally follow the differentiation and attempt all given problems. This action by students results from Korean parents fervor for education and the lack of linkage between the differentiation and evaluation system. Thus, slower learners feel anxiety unless they study the same material as more advanced learners. The teachers had differing opinions on suitable learning quantity and whether it was appropriate to continue with the differentiated curriculum.

Second, agreeing that emphasize on active student participation in problem solving, the teachers had difficulty confronting the shortage of additional materials or learning manipulatives to proceed with the programmed as constructed. In those cases, the full meaning of concepts or operations was not thoroughly covered, according to the interviews. Also, since too much time was spent solving tasks, teachers seemed to have trouble planning or proceeding with lessons.

Third, using real-life problem situation contexts caused students to suffer from complex computations since data in real life are not simple. It would not have been much of a problem if a calculator or computer could have been used adequately and easily, but most Korean secondary schools still are not satisfied with these standards.

Fourth, the teachers believed that the number of problems that provided chances to think mathematically out numbered those for drill practice in the new textbooks. Those problems helped teachers guide students toward a more meaningful and powerful learning experience. However, the teachers emphasized that class size reduction and the gap between higher and lower achievers in Korean secondary schools still need to be addressed.

Lastly, the new textbooks provided a variety of materials and tasks related to other subjects, which enriched mathematics classes. Sometimes spontaneous study groups came into being among students to develop mathematical ideas discovered in the questions and tasks of the textbook.
VI. Concluding Remarks

The review of KSMTs reveals that authors tried to embody national curriculum reform ideas, which can be summarized as focusing on differentiation and optimization in educational contents and instruction. In particular, a lot of new mathematical context was used to encourage exploration, and problems at differing skill levels were presented for regardless of differences between students' ability and attitude. Active adaptation from famous foreign mathematics textbooks and intention to keep the characteristics of Korean textbooks seem to be embedded in the textbooks. However, the teachers who participated in the interviews pointed out several problems with the textbooks. Textbook improvement has been one of the most important research themes that the MOE supports. More extensive and in-depth investigation of Korean mathematics textbooks needs to be conducted so that the progression in textbooks can really affect a lesson.

References


Gang Sub, Lee et al. (2002). Differentiation and Integration. Seoul, Korea: Jihak Textbooks.


Jeong Ho, Woo et al. (2002). Differentiation and Integration. Seoul, Korea: Daehan Textbooks.


Sung Eun, Go et al. (2002). Mathematics 9-B. Seoul, Korea: BlackBox Textbooks.

Tae Gun, Jo et al. (2000). Mathematics 7-B. Seoul, Korea: Gumseong Textbooks.


Young Ha, Lee et al. (2000). Mathematics 7-B. Seoul, Korea: Gyomoonsa Textbooks.