2010년 Matrix Theory Final, June 1st 11:45AM ~7th 11:00AM (Submit)

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V. Self Evaluation (50pts)(별지) - 파일로 제출.

참여 확인과 본인의 Project (Term paper) Proposal 에 대해 아래를 채우시오.

1. 본인이 그간 Q&A, 동료학생, "행렬론" 강좌등에 기여한 내용을 간단히 서술하세요!
 (1) Q&A 참여 개인 total 약 (55) 회 (스스로 QnA 에서 검색하여 확인 가능)

2. 자신이 한 학기 동안 PBL-BL English MT 강좌에서 학습한 내용을 나름대로 모두 정리하여 제출하세요.

1. Linear Equations and Matrices.

Information in mathematics is often represented into rows and columns to form rectangular arrays. And that form said to be "matrices". We can learn in this chapter appropriate operations on the matrices. In this section we discussed a method for solving some systems. The basic method for solving a system of linear equations is to replace the given system by a new system.

Although the given system is changed by these operations, the new system has same solution. These three operations are called elementary row operations. Using these operations, we can find solution easier. And then we learned RREF and REF. These procedures are based on the idea of reducing augmented matrix of a system. We can distinct theses two procedure. Producing a matrix in RREF is called Gauss-Jordan elimination, and producing a matrix in REF is called Gaussian elimination. Important thing in this chapter is LDU factorization. LDU factorization is simply factoring matrix into a lower triangular and upper triangular. The advantage of this method is well suited for computers and many computer programs. I think this chapter may seem to be easy, but actually not simple.

2. Determinant.

In this chapter, we learned the concept and properties of determinant. Determinant function has important applications to the theory of linear system. By determinant test, we can find invertible matrix. And this time we become understand very important list. Equivalent statements are very useful to solve or prove some problems. And using Cramer's rule, we can solve a linear system. This rule is useful for studying properties of a system without the need for solving system.

3. Vector Space.

Definition of vector must satisfy eight rules. Simply there are the scalar multiplication and the sum. We must keep in mind this concept. And there are other concepts in this chapter. The concept of subspace, linear transformation, basis, linear (in)dependent, rank and nullity are important, too. According to linearly independent, there are many theorems. For example, let $S=\{\mathbf{v}_1,\mathbf{v}_2,\dots,\mathbf{v}_r\}$ be a set of vectors in$\mathbb{R}^n$. If $r\geq n$, then S is linearly dependent. We can prove this theorem using homogeneous system of $n$equations in the r unknowns.

Actually I learned many things through proving this problem.

P.115 Exercises3.23

Determine whether the fooling statement are true or false, and justify your answer.

(1) The set of all $n\times n$ matrices A such that $A^T=A^{-1}$ is a subspace of the vector space $M_{n\times n}(R)$.
False

(2) If α and β are linearly independent subset of a vector space V, so is their union $\alpha\cup\beta$. False

(3) If U and W are subspaces of a vector space V with bases α and β respectively, then the intersection $\alpha\cap\beta$ is basis for $U\cap W$. False

(4) Let U be the row-echelon form of a square matrix A. If the first r columns of U are linearly independent, so are the first r columns of A. True

(5) Any two row-equivalent matrices have the same column space. True

(6) Let A be an $m\times n$ matrix with rank m. Then the column vectors of A span R^m. True

(7) Let A be an $m\times n$ matrix with rank n. Then $A\mathbf{x}=b$ has at most one solution. True

(8) If U is a subspace of V and $\mathbf{x},\mathbf{y}$ are vectors in$V$ such that $\mathbf{x}+\mathbf{y}$ is contained in$U$, then $\mathbf{x}\in U$, $\mathbf{y}\in U$. False

(9) Let U and V are vector spaces. Then U is a subspace of V if and only if $\dim U\leq\dim V$. False

(10) For any $m\times n$ matrix A, $\dim C(A^T)+\dim N(A^T)=m$. True

(Sol)

(1) The set of all $n\times n$ matrices A such that $A^T=A^{-1}$ is a subspace of the vector space $M_{n\times n}(R)$.

Let $V=\{A\in M_{n\times n}(R)\,|\,A^{-1}=A^T\in M_{n\times n}(R)\}$.

And let $A,B\in V$.

[Check the subsets are closed under the vector addition and scalar multiplication.]

First:

$(A+B)^{-1}\neq A^{-1}+B^{-1}=A^T+B^T=(A+B)^T$

$\therefore A+B\in V$

Second:

$(kA)^{-1}=\frac{1}{k}A^{-1}$ but,

$(kA)^T=kA^T$.

$\therefore kA\not\in V$

Therefore, the statement (1) is false. ■

(4) Let U be the row-echelon form of a square matrix A. If the first r columns of U are linearly independent, so are the first r columns of A.

Elementary row operations do not change the row space of a matrix and the nullspace of a matrix.

But, Elementary row operations can change the column space of matrix.

Suppose a matrix U results from performing an elementary row operation on an $n\times n$ matrix A.

Then the two homogeneous linear system,

$A\mathbf{x}=0$ and $U\mathbf{x}=0$ have the same solution set.

Thus the first system has a non-trivial solution if and only if the same true of the second. But if the column vectors of A and U, respectively, are

$c_1,c_2,\dots,c_n$ and $c_1{}',c_2{}',\dots,c_n{}'$.

Then the two system can be rewritten as

$$x_1c_1+x_2c_2+\cdots+x_nc_n=0 \qquad (1)$$

and $$x_1c_1{}'+x_2c_2{}'+\cdots+x_nc_n{}'=0. \qquad (2)$$

Thus (1) has a non-trivial solution for $x_1,x_2,\dots,x_n$ if and only if the same is true of (2).

This implies that the column vectors of A are linearly independent if and only if the same is true of U.

So we have the following result.

[If A and B are row equivalent matrices, then a given set of column vectors of A is linearly independent if and only if the corresponding column vectors of B are linearly independent.]

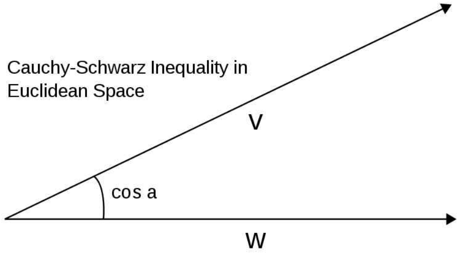
Therefore, the statement (4) is true. ■

(10) For any $m \times n$ matrix A, $\dim C(A^T) + \dim N(A^T) = m$.
Since $\dim C(A^T) = \dim R(A) = \dim C(A)$, we can get
$\dim C(A^T) + \dim N(A^T)$
$= \dim C(A) + \dim N(A^T) = m$ (By Rank Theorem)
$\therefore \dim C(A^T) + \dim N(A^T) = m$
Therefore the statement (10) is true. ■

Above problems help me more understand in matrix theory.
Moreover, the equivalent statements in preceding chapter are complemented. That statements is really important key for proving theorems.
Through studying this chapter, I learned the matrix theory more deep.

4. Linear Transformation.
Linear transformation from one vector space to another has important applications in physics, engineering and various fields in science. In this chapter, we learned basic properties of linear transformation. Personally my most interest part of this lecture is linear transformation. We can find the characteristic of vector spaces through the concept of kernel of nullity. Also we can study the equivalent statements more. For instance, linear transformation is one-to-one if and only if the kernel of transformation is 0, or if and only if the nullity if transformation is 0. There exist other amazing properties according to linear transformation. I think these properties are awesome.

5. Inner product Spaces.
In this chapter, we learned a very simple and important theorem. That is Cauchy-Schwarz inequality. Marvelous Cauchy-Schwarz inequality holds if and only if the vectors are linearly dependent. We can search so many data about this rule on internet.



This image can explain the principle of Cauchy-Schwarz inequality simply.
We can also learn projection and Gram-Schmidt process. Later we can find the solution of quadratic form through this process.

6. Diagonalization.
Given $n \times n$ matrix A, does there exist a basis for R^n consisting of eigenvectors of A?
Given $n \times n$ matrix A, does there exist an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix?
These two problems seem different, but actually same. Thus, A has n linearly independent eigenvectors if and only if A is diagonalizable.

So many theorems and definitions are logic. All of them is connected each other.
And these concepts of diagonalization and eigenvectors are used in many areas like economics, geometry.

7. Complex Vector Space.
In this chapter we develop the basic properties of vector spaces with complex scalars.

8. Jordan Canonical Forms.
in this section, mathematic is very useful. Some calculation is too complex, so spend many time.
Jordan Canonical Forms is special type of block matrix in which each block consists of Jordan blocks.

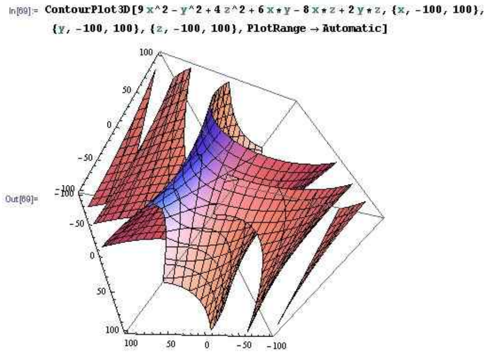
9. Quadratic Forms.
Studying this chapter, I become proficient at Mathematica.
When I solving problem like this, Mathematica is really useful.

P323. Problem 9.1
Find the symmetric matrices representing the quadratic form.
(1) $9x_1^2 - x_2^2 + 4x_3^2 + 6x_1x_2 - 8x_1x_3 + 2x_2x_3$,
(2) $x_1x_2 + x_1x_3 + x_2x_3$,
(3) $x_1^2 + x_2^2 - x_3^2 - x_4^2 + 2x_1x_2 - 10x_1x_4 + 4x_3x_4$.

[Solution]

(1) $9x_1^2 - x_2^2 + 4x_3^2 + 6x_1x_2 - 8x_1x_3 + 2x_2x_3 = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 9 & 3 & -4 \\ 3 & -1 & 1 \\ -4 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\therefore \begin{bmatrix} 9 & 3 & -4 \\ 3 & -1 & 1 \\ -4 & 1 & 4 \end{bmatrix}$



(2) $x_1x_2+x_1x_3+x_2x_3=[x_1\ \ x_2\ \ x_3]\begin{bmatrix}0 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$

$$\therefore \begin{bmatrix}0 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}$$

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In[71]:= ContourPlot3D[x*y + x*z + y*z, {x, -1000, 1000}, {y, -1000, 1000},
    {z, -1000, 1000}, PlotRange → Automatic]
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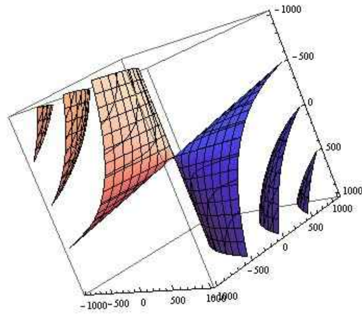
Out[71]=



Quadratic form also has many applications. To solve the quadratic form problem, we use the concepts of preceding chapters. We have to find eigenvalues, diagonalize the matrix, and find unit vectors by Gram-Schmidt process.

3. (1) 본인이 PBL-BL English MT 강좌를 통하여 배운 수학적 내용 중 특히 기억나는 내용을 서술하시오

To solve Exercises3.23 and Exercises4.32, I spend lot of time thinking about it.
Although that statements are simple, but it was a challenge.

(2) 동료와 같이 MT 1-9장을 cover 하면서 배우거나 느낀 점은?

This semester when I was learning Matrix theory, I found out that this subject is pretty much hard. Actually, I realized that mathematics is not easy at all. I am a senior who is looking for graduation very soon, but still I find some unfamiliar orthography and proving problems. Usually mathematics is thought as logical studies. But what happens when others does not understand my own proving progresses? The progress might be useless, if I cannot make others understand my knowledge easy.
We are living in high-technology world. One can read research papers from other countries online, and exchange information at real time. By the world-wide-web, people exchange enormous amount of information that one cannot imagine. For instance, if one searches Gram-Schmidt process on Google, one can find not only definition of it but also examples and a lot of information about it.

Most of all, I think creating correct information is more important than searching correct information. In this class I learned how to make correct information, and realized that mathematics is not listing of facts it is logical studies. I appreciate my professor to give us this useful information during this semester.

자 기 평 가 (Midterm) 1

과 목 명	MT 행 렬 론	조	조
이 름		전 공	수학과

평가항목	전혀 아니다	아니다	약간 아니다	약간 그렇다	그렇다	매우 그렇다
1. 출석 및 시간을 지켰다.						v
2. QnA 및 토론에 적극적으로 참여하였다.					v	
3. 토의내용에 적합한 질문과 응답을 하였다.					v	
4. 동료에게 도움이 되는 질문, 답, 정보를 제공하였다.						v
5. 다른 동료의 의견을 존중하였다.						v
6. 문제 관련 토론의 조직·운영 및 의견수렴과정에 긍정적으로 기여하였다.						v
7. 같은 조의 조원들이 나와 같이 활동하고 싶어 한다.					v	

강좌 관련 개선의견

QnA 외에 다른 게시판을 만들어서 문제토론과 쉬어가는 문제들을 구분했으면 합니다. 예를 들어 스토쿠 같은 숫자 퍼즐이나, 수학관련 영화 자료, 책자료 같은 것들을 업로드 했으면 좋지 않았을까 싶습니다.

자 기 평 가 (Midterm) II

과 목 명	MT 행렬론	조	5조
이 름		날 짜	
학습문제	MT PBL 자기주도적 수업, 자기 성찰노트		

자기 점검표

활동(Activity)	Excellent	Good	Fair
1. 나는 문제해결에 필요한 아이디어와 사실들을 생성하는데 기여하였다.		v	
2. 나는 학습과 관련된 학습과제(Learning issue:더 알아야 할 사실들)들을 제안하였다.		v	
3. 나는 개인학습을 할 때 다양한 학습 자료를 사용하였다.	v		
4. 나는 새로운 정보와 지식제공에 기여하였다.	v		
5. 나는 문제 제기와 토의에 적극적으로 참여하였고 토의의 촉진과 이해를 위한 적절한 질문을 많이 제공하였다.	v		
6. 나는 우리 조가 원활한 조 활동을 하는데 기여하였다.	v		

[성찰노트] ※ 다음 각각의 사항에 대하여 자신의 활동내용을 기록하세요.

관련된 다양한 문제들을 다뤄보기 위해 우리 교과서 뿐 아니라 다른 책들도 많이 찾아보았습니다. 이번 행렬론 수업을 들으면서 밤도 많이 새고 그만큼 얻는 것도 많았던 것 같습니다.
http://www.mathhelpforum.com/math-help/
이 홈페이지는 서로 각 분야에 대해 질문하고 그 질문에 대한 답을 아는 사람은 답글을 달아 서로 토론하는 곳입니다. 우리 행렬론 QnA의 국제판이라고 할 수 있는데요., 이곳에서 많은 아이디어도 얻었고 자극도 받았습니다. 거기서 받은 자극을 행렬론 공부하면서 함께 나누려 했으니 부족했던 것 같습니다. 조장으로서 문제풀이를 많이 하려고 노력했습니다. 9장 같은 경우는 한문제당 푸는 시간이 꽤 걸려서 힘들었지만 그래도 그만큼 머릿속에 확실히 남아 흐뭇합니다.
조원들과는 온라인상에서도 만나며 많은 이야기를 나누었습니다.