Spring 2013, Calculus I, Final Comprehensive Exam					Sign
Course	**Calculus I** with Sage	GEDB020-42	Prof.	Sang-Gu Lee	

Major		Year(학년)		S.N(학번)		Name	

※ Notice (수험생 유의사항)

1. Write your name and get the signature. 답안 작성전에 이름 등을 빠짐없이 기입하고 감독자 날인을 받으세요.
2. Keep your Honor code. If not, serious penalty will be given.

Score (150)	
Final-PBL	EXAM
/50	/100

http://matrix.skku.ac.kr/cal-lab/sage-grapher.html , http://matrix.skku.ac.kr/cal-lab/sage-grapher-para.html

var('a, b, c, d')	# 변수정의	integral(f(x), x)	# 부정적분
limit(f(x), x=a)	# 극한	integral(f(x), x, a, b)	# 정적분
limit(f(x), x=a, dir='minus')	# 좌극한	plot(f(x), (x, a, b)	# 함수의 그래프
limit(f(x), x=a, dir='plus')	# 우극한	implicit_plot(f, (x, a, b), (y, c, d))	# 음함수 그래프
limit(f(x), x=+oo)	# 무한대에서의 극한	find_root(f(x), a, b)	# 근사해 구하기
limit(f(x), x=-oo)		var('t')	# 변수정의 (매개변수방정식)
solve(f(x)==0, x)	# Solve 방정식 풀이	x=2+2*t	
diff(f(x), x)	# 도함수	y=-3*t-2	
diff(f(x), x, 2)	# 2계 도함수	parametric_plot((x, y), (t, -10, 10), rgbcolor='red')	# 직선 Plot
		f.partial_fractions(x)	# 부분분수

I. (2pt x 9 = 18) Mark True(T) or False(F) in the blanks ( ).

1. ( ) If $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exist, then $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$ also exists.

2. ( ) The most famous application of the use of hyperbolic cosine is to describe the shape of a hanging wire. If a heavy flexible cable (such as a telephone or power line) is suspended between two points at the same height, then it takes the shape of **a catenary curve** with equation $y = c + a\cosh(x/a)$, which uses the hyperbolic cosine. (The Latin word catena means "chain".)

3. ( ) Let $C(x)$ be the **cost function** of producing x units of a certain product. Then the **marginal cost** is the rate of change of $C(x)$ with respect to x, that is, the marginal cost function is $\dfrac{C(x)}{x}$. The marginal cost function represents the cost per unit when x units are produced.

4. ( ) (Example 7-4-7) We always have the form of the **partial fraction** decomposition of the function as following;

$$\frac{x^4+x^3+x^2+x+2}{(x+1)(x-2)(x^2-x+4)^4} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{Cx+D}{(x^2-x+4)^4}.$$

5. ( ) Consider an object moving along a straight line with position function $s(t)$. Then the **force** F on the object (in the same direction) is $F = m\dfrac{d^2s}{dt^2}$. And **the work** done is defined to be the product of the force F and the distance d that the object moves.

6. ( ) The **average value** of f on the interval $[a, b]$ is defined as $f_{\text{ave}} = \dfrac{1}{a-b}\displaystyle\int_a^b f(x)dx$

7. ( ) The **Simpson's Rule** gives a much better approximation to the exact value of the integral than does the Trapezoidal Rule or the Midpoint Rule. The **Trapezoidal Rule** gives a better approximation to the exact value of the integral than does the **Midpoint Rule**.

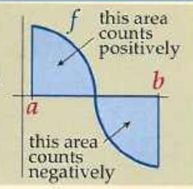
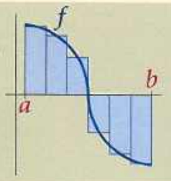
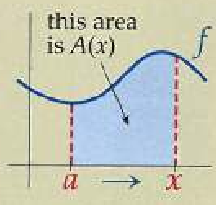
8. ( ) **The area of the surface of revolution** obtained by rotating the curve $y = f(x)$, $a \le x \le b$, about the x-axis is given by $S = \displaystyle\int_a^b 2\pi f(x)\sqrt{1-[f'(x)]^2}\,dx$

9. ( ) Thus the **arc length** $L = \displaystyle\int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\,d\theta$ of the polar curve is $L = \displaystyle\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$.

Ⅱ. (3pt x 4 = 12) State or Define.

1. **Choose 4** terminology or concept and **state its meaning** as much as you knows **in your words.**

① Newton's Method ② Fundamental Theorem of Calculus ③ Definite Integral ④ Indefinite integral ⑤ Improper Integral (이상적분)
⑥ Integration by parts ⑦ Volume of a solid of revolution ⑧ The Arc Length Formula ⑨ Comparison Theorem for integral
⑩ Volume of a solid S that lies between $x=a$ and $x=b$ ⑪ The method of cylindrical shells

	Vocabulary	Intuition	Mathematics
(1)	$\int_a^b f(x)\,dx$, the **definite integral** of f on $[a, b]$	the signed area between the graph of f and the x-axis from $x=a$ to $x=b$ 	$\int_a^b f(x)dx = \lim_{N\to\infty}\sum_{k=1}^{N} f(x_k^*)\Delta x$, where $\Delta x = \frac{b-a}{N}$, $x_k = a + k\Delta x$, and $x_k^* \in [x_{k-1}, x_k]$ 
(2)	$\int f(x)\,dx$, the **indefinite integral** of f	$\int f(x)\,dx$ is the family of antiderivatives of $f(x)$ (i.e. the set of functions whose derivatives are $f(x)$)	$\int f(x)\,dx = F(x) + C$, where $F(x)$ is any antiderivative of $f(x)$ (i.e. $f'(x) = f(x)$)
(3)	the **area accumulation function** for f on $[a, b]$	for $x \in [a, b]$, $A(x)$ = signed area between the graph of f and the x-axis from a to x 	for $x \in [a, b]$, $A(x) = \int_a^x f(t)dt$

▶▶ **Sol**

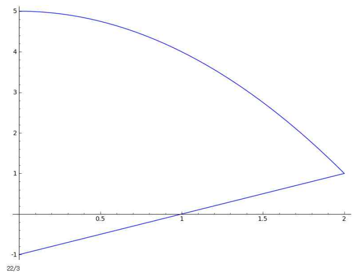
Ⅲ. (5pt x 8 = 40pt) Find or Explain or Fill the blanks ☐ .

1. (MidTerm) Use differential to approximate $\sqrt{98}$.

▶▶ **Sol** Let $f(x)=\sqrt{x}$. Set $x=100$ and $\Delta x=-2$. Since $dx \approx \Delta x$, $df=\dfrac{dx}{2\sqrt{x}}$, we have $df|_{x=100} = -\dfrac{1}{10}$.

2. (Sage) Sketch the region enclosed by $y=x-1,\ y=5-x^2,\ x=0,\ x=2$ and find its area

```
f(x)=x-1
g(x)=5-x^2
show(plot(f(x), x, 0, 2) + plot(g(x), x, 0, 2))
print integral(                                  )
```



3. Verify the formula $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right)+C$ by Sage .

Proof) $x=a\sin\theta$, $dx = a\cos\theta$ $\qquad\theta\in[-\frac{\pi}{2},\frac{\pi}{2}]$

$$\int \sqrt{a^2-x^2}\,dx = \int a^2\cos^2\theta\, d\theta = \frac{a^2}{2}(\theta+\sin\theta\cos\theta)+C = \frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right)+C$$

Verify the above formula by Sage

▶▶ **Sol**

```

```

1/2*a^2*arcsin(x/sqrt(a^2)) + 1/2*sqrt(a^2 - x^2)*x # $\frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right)+C$

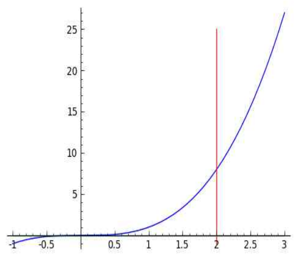
=> RHS= LHS ■

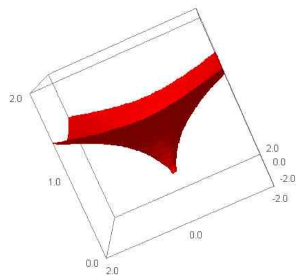
4. Calculate the volume of the solid obtained by rotating the region bounded by curve $y=x^3$, $x=2$, $y=0$ about the x-axis.

▶▶ **Sol** $A(x) = \pi(x^3)^2$, $V = A(x) = \frac{128}{7}\pi$

(Sage)

```
var('x')
f(x)=
print integral(                    )
```





128/7*pi # $\frac{128}{7}\pi$

5. A bucket that weighs 5 lb and a rope of negligible weight are used to draw water from a well that is 90 ft deep. The bucket is filled with 30 lb of water and is pulled up at a rate of 3 ft/s, but water leaks out of a hole in the bucket at a rate of 0.4 lb/s. Find the work done in pulling the bucket to the top of the well.

Sol $W = Fd = mgh$

$\frac{dW}{dt} =$ [   ] (by Chain rule)

and since mass and height changes over time as given:

$m = 35 - 0.4t, \quad h = 90 - 3t \quad \Rightarrow \frac{dW}{dt} =$ [   ]

Integrate by t from 0 to 30, and substitute the values to get the total work done:

$W = 32 \times \int_0^{30}$ [   ] dt

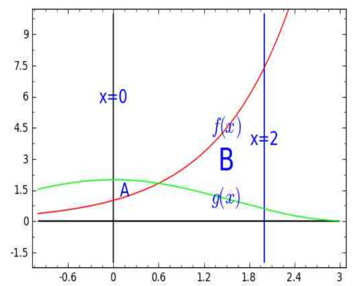
$\therefore W = 32\,[69t]_0^{30} = 66240$ (in ft·lb) ■

6. Calculate the total area of A part and B part marked in the graph below. Here $f(x) = e^x$ and $g(x) = \cos(x) + 1$.

Sol (Sage)

```
var('x')
f(x)=exp(x)
g(x)=cos(x)+1
x1=find_root(f(x)==g(x),0,2)
a=integral(                    )
b=integral(                    )
a+b
```

e^2 - 3.22424574744



7. (Exs 7-2-5) Write how you are going to show a formula $\int \sin^5 x \cos^{-6} x\, dx = \frac{1}{5}\sec^5 x - \frac{2}{3}\sec^3 x + \sec x + C$.

Sol (Either by hand or Sage)

Sage를 이용하여 공식을 증명하려는 경우는 어떤 과정을 거치는지와 **명령어만 서술하면 됩니다.**)

1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5 $= \frac{1}{5}\sec^5 x - \frac{2}{3}\sec^3 x + \sec x + C$

8. Find the length of the curve $y^2 = x$ from $(0,\ 0)$ to $(1,\ 1)$.

▶▶ **Sol** Since $2y\,dy = dx$ implies $\frac{dx}{dy} = 2y$,

$$L = \int_0^1 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \boxed{} = 2\int_0^1 \sqrt{y^2 + \frac{1}{4}}\, dy$$

Let $y = \frac{1}{2}\tan\theta$, then $dy = \frac{1}{2}\sec^2\theta\, d\theta$ and $\sqrt{y^2 + \frac{1}{4}} = \frac{1}{2}\sec\theta$.

$y \to 0 \quad \Rightarrow \quad \theta \to 0, \qquad y \to 1 \quad \Rightarrow \quad \theta \to \tan^{-1}2$

$\tan\theta = 2,\ \sec\theta = \sqrt{\tan^2\theta + 1} = \sqrt{5}$

$$L = 2\int_0^1 \sqrt{y^2 + \left(\frac{1}{2}\right)^2}\, dy = \boxed{} = \frac{1}{2}\int_0^{\tan^{-1}2} \sec^3\theta\, d\theta$$

$$= \frac{1}{4}\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right]_0^{\tan^{-1}2} = \frac{1}{4}\{2\sqrt{5} + \ln(\sqrt{5}+2)\}$$

$$= \frac{\sqrt{5}}{2} + \frac{\ln(\sqrt{5}+2)}{4} \quad ■$$

IV. (6pt x 5 = 30pt) Prove or Explain (Fill the blanks $\boxed{}$).

1. Show $\lim_{x \to -1} x^2 + 2x - 8 = -9$ using $\epsilon - \delta$ method.

▶▷ **Proof :**

$\forall \epsilon > 0$ [Find δ] Let $\delta =$

[Side Calculation] $f(x) - f(-1) = x^2 + 2x - 8 + 9 = x^2 + 2x + 1 = (x+1)^2$ ■

2. Show $\frac{d}{dx}(\text{sech } x) = -\text{sech } x \tanh x$.

▶▷ **Proof :**

$$\frac{d}{dx}(\text{sech } x) = \left(\frac{1}{\cosh x}\right)' = \boxed{} = -\tanh x\, \text{sech } x \quad ■$$

3. Find $\int_{-\infty}^{-1} x e^{-x^2} dx$ (You may use Sage in part, but I recommend you to solve it **by hand**.)

▶▷ **Sol :**

$$\int_{-\infty}^{-1} x e^{-x^2} dx = \lim_{t \to -\infty} \int_t^{-1} x e^{-x^2} dx =$$

$$= -\frac{1}{2e}. \quad ■$$

4. **Verify** $\int \frac{dx}{x^3+1} = \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{\sqrt{3}(2x-1)}{3}\right) + \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1)$ **by Sage** .

Proof) $\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ => $A = \frac{1}{3}, B = -\frac{1}{3}, C = \frac{2}{3}$

$$\int \frac{dx}{(x+1)(x^2-x+1)} = \frac{1}{3}\int \frac{1}{x+1} - \frac{x-2}{x^2-x+1}dx = \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{6}\int \frac{2x-1}{x^2-x+1}dx + \frac{1}{6}\int \frac{3}{x^2-x+1}dx$$

$$= \frac{1}{3}\int \frac{1}{x+1}dx - \frac{1}{6}\int \frac{2x-1}{x^2-x+1}dx + \frac{1}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}}dx = \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{2}\left[\sqrt{\frac{4}{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right] + C$$

$(\because \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C)$

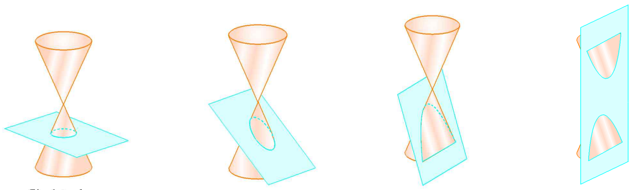
Verify the above formula by Sage

Sol

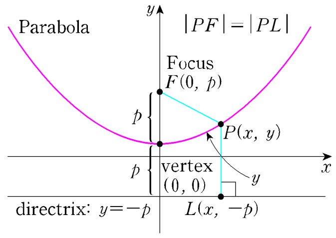
1/3*sqrt(3)*arctan(1/3*(2*x - 1)*sqrt(3)) + 1/3*log(x + 1) - 1/6*log(x^2 - x + 1)

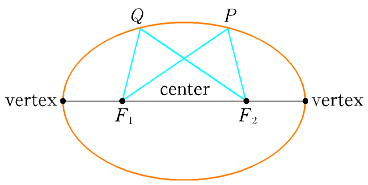
=> RHS= LHS ■

5. **(Conic section)** A conic section (or just conic) is a curve obtained by intersecting a cone (more precisely, a right circular conical surface) with a plane. The three types of conic section are the hyperbola, the parabola, and the ellipse. The circle is a special case of the ellipse. **Sketch (state) what you know about the process of forming the formulas of hyperbola, parabola, and ellipse.**

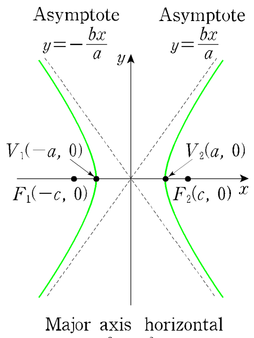


Circle: plane perpendicular to cone axis | Ellipse: plane cuts one half of cone | Parabola: plane parallel to side of cone | Hyperbola: plane cuts both halves of cone





$|PF_1|+|PF_2|=|QF_1|+|QF_2|$



Major axis horizontal

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Derivatives of Inverse Trigonometric Functions[1]

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

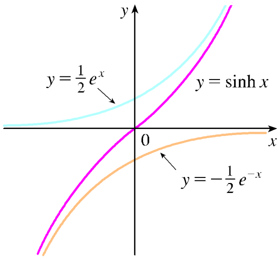
Definition of the Hyperbolic Functions[2]

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

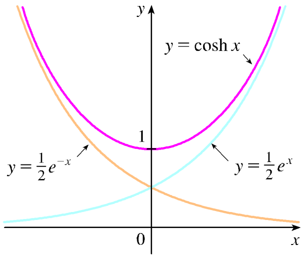
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\text{sech}\, x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} \qquad \text{csch}\, x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

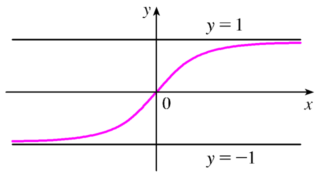
The graphs of the hyperbolic sine and cosine can be sketched using graphical addition as in Figures 8 and 9.



$y = \sinh x = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$



$y = \cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$



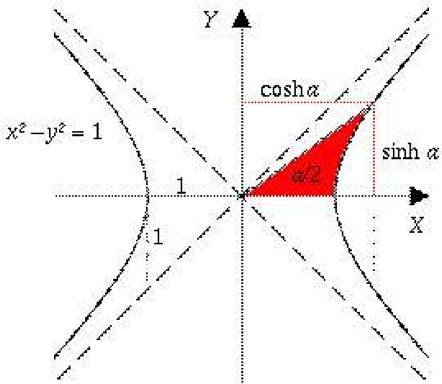
$y = \tanh x$

Derivatives of the Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\coth x) = -\text{csch}^2 x$$

$$\frac{d}{dx}(\cosh x) = \sinh x \qquad \frac{d}{dx}(\text{sech}\, x) = -\text{sech}\, x \tanh x$$

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x \qquad \frac{d}{dx}(\text{csch}\, x) = -\text{csch}\, x \coth x$$



Relationship to the exponential function

Definition of the Inverse Hyperbolic Functions

$\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$	$(x \in R)$
$\cosh^{-1} x = \ln(x + \sqrt{x^2-1})$	$(x \geq 1)$
$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$	$(\lvert x\rvert < 1)$
$\coth^{-1} x = \frac{1}{2}\ln\frac{x+1}{x-1}$	$(\lvert x\rvert > 1)$
$\text{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$	$(0 < x < 1)$
$\text{csch}\, x = \ln\left(\frac{1}{x} + \frac{\sqrt{1-x^2}}{\lvert x\rvert}\right)$	$(x \neq 0)$

Derivatives of the Inverse Hyperbolic Functions

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$	$(x \in R)$
$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$	$(x > 1)$
$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$	$(\lvert x\rvert < 1)$
$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$	$(\lvert x\rvert > 1)$

The volume of the solid obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b, is

$$\int_a^b (2\pi x) \quad [f(x)] \quad dx \qquad \text{where} \qquad 0 \leq a < b\ .$$

(circumference) · (height) · (thickness of shell).

1) http://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/invtrigderivdirectory/InvTrigDeriv.html
2) http://en.wikipedia.org/wiki/Hyperbolic_function

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ $\int \frac{1}{x} dx = \ln|x| + C$ $\int e^x dx = e^x + C$ $\int a^x dx = \frac{a^x}{\ln a} + C,\ a > 0$

$\int \sin x\, dx = -\cos x + C$ $\int \cos x\, dx = \sin x + C$ $\int \sec^2 x dx = \tan x + C$ $\int \csc^2 x\, dx = -\cot x + C$

$\int \sec x \tan x\, dx = \sec x + C$ $\int \csc x \cot x\, dx = -\csc x + C$ $\int \tan x\, dx = \ln|\sec x| + C$

$\int \cot x\, dx = \ln|\sin x| + C$ $\int \sinh x\, dx = \cosh x + C$ $\int \cosh x\, dx = \sinh x + C$

$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C,\ |x| \le |a|$ and $a > 0$ $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C,\ |x| \ge a > 0$

$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C,\quad a \neq 0$ $\int \frac{1}{(x-a)^n} dx = \begin{cases} -\frac{1}{(n-1)(x-a)^{n-1}} & (n \neq 1) \\ \ln|x-a| & (n = 1) \end{cases}$

$$\int \frac{x}{(x^2 + a^2)^n} dx = \begin{cases} -\frac{1}{2(n-1)(x^2 + a^2)^{n-1}} & (n \neq 1) \\ \frac{1}{2}\ln|x^2 + a^2| & (n = 1) \end{cases}$$

Table of Integration Formulas [Constant of integration is omitted.]

1. $\int x^n dx = \frac{x^{n+1}}{n+1}\ (n \neq -1)$ 2. $\int \frac{1}{x} dx = \ln|x|,\ x \neq 0$ 3. $\int e^x dx = e^x$ 4. $\int a^x dx = \frac{a^x}{\ln a},\ a > 0$

5. $\int \sin x\, dx = -\cos x$ 6. $\int \cos x\, dx = \sin x$ 7. $\int \sec^2 x\, dx = \tan x$ 8. $\int \csc^2 x\, dx = -\cot x$

9. $\int \sec x \tan x\, dx = \sec x$ 10. $\int \csc x \cot x\, dx = -\csc x$ 11. $\int \sec x\, dx = \ln|\sec x + \tan x|$

12. $\int \csc x\, dx = \ln|\csc x - \cot x|$ 13. $\int \tan x\, dx = \ln|\sec x|$ 14. $\int \cot x\, dx = \ln|\sin x|$

15. $\int \sinh x\, dx = \cosh x$ 16. $\int \cosh x\, dx = \sinh x$ 17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right),\ a \neq 0$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right),\ a \neq 0$ 19. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|,\ a \neq 0$ 20. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}|$

Midpoint Rule

$\int_a^b f(x)dx \approx M_n = \Delta x[f(\overline{x_1}) + f(\overline{x_2}) + \cdots + f(\overline{x_n})]$ where $\Delta x = \frac{b-a}{n}$ and $\overline{x}_i = \frac{1}{2}(x_{i-1} + x_i) =$ midpoint of $[x_{i-1},\ x_i]$.

The Trapezoidal Rule

$\int_a^b f(x)dx \approx T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$ where $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$.

http://matrix.skku.ac.kr/cal-lab/Area-Sum.html

(Composite) Simpson's Rule

$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$

where n is even and $\Delta x = (b-a)/n$.

4 **Error Bound for Simpson's Rule**

Assume that $|f^{(4)}(x)| \le K$ for $a \le x \le b$. Let E_S be the error involved in using Simpson's Rule, then

$$|E_S| \le \frac{K(b-a)^5}{180n^4}.$$

The symbol $\int_{-\infty}^{b} f(x)dx$ is defined by the equation $\int_{-\infty}^{b} f(x)dx = \lim_{t \to -\infty} \int_t^b f(x)dx$.

The improper integrals $\int_a^{\infty} f(x)dx$ and $\int_{-\infty}^{b} f(x)dx$ are called **convergent** if the corresponding limit exists; otherwise they are called **divergent**.

The rotation about the x-axis the surface area is $S = \int 2\pi y \, ds$

The rotation about the y-axis, the surface area formula is $S = \int 2\pi x ds$

Theorem

A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \quad \text{or} \quad r = \frac{ed}{1 \pm e\sin\theta}$$

represents a conic section with eccentricity e, where $|d|$ is distance between the focus oat the pole and its corresponding directrix.

Theorem Pappus's centroid theorem (The second)

The volume V of a solid of revolution generated by rotating a plane figure F about an external axis is equal to the product of the area A of F and the distance d traveled by its geometric centroid.

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$$V = Ad.$$

Formula for the area A of the polar region R is $A = \int_a^b \frac{1}{2}[f(\theta)]^2 d\theta$. Since $r = f(\theta)$,

it may be written as $A = \int_a^b \frac{1}{2} r^2 d\theta$.

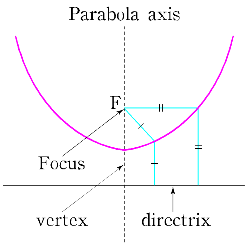

Figure 2

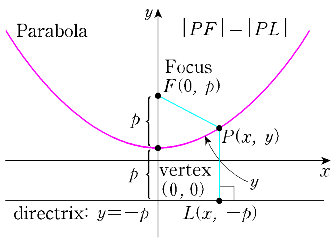

Figure 3

$|PF| = \sqrt{x^2 + (y-p)^2}$, $\sqrt{x^2 + (y-p)^2} = |y+p|$. $x^2 + (y-p)^2 = |y+p|^2 = (y+p)^2$

$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$, $\quad x^2 = 4py$.

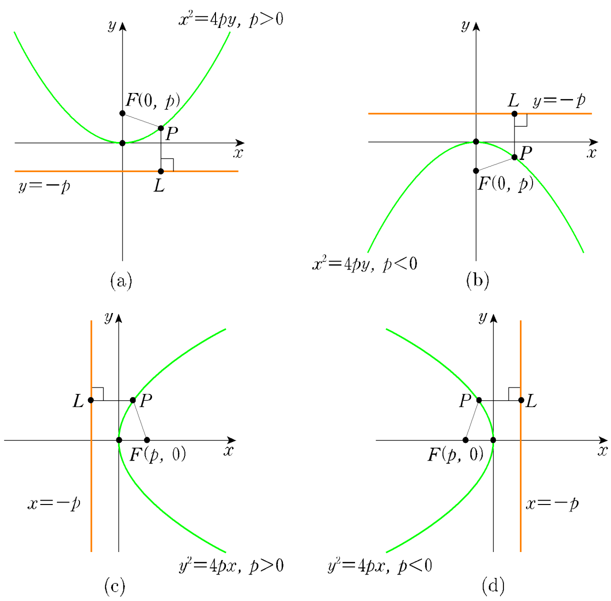

Figure 4

미적분의 관계도 by SGLee