Chapter 3
Development and Characteristics of Korean Elementary Mathematics Textbooks

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Abstract
Korean mathematics education reform centers around revision of the national curriculum and concomitant textbooks with teachers’ guidebooks. Almost all Korean teachers use them as their main instructional resources. Given this, it is important to probe Korean mathematics textbooks so as to better understand Korean mathematics education. This paper presents general flows and directions in developing textbooks and related materials, followed by their main characteristics along with examples. This paper also deals with some issues on the future development of mathematics textbooks.

I. Introduction
The development of elementary mathematics textbooks and their related resources such as students' workbooks and teachers' guidebooks is tightly related to the change of the national mathematics curriculum. In fact, Korean mathematics education reform centers around revising the national curriculum and concomitant textbooks with teachers’ guidebooks. Whereas educational leaders in Korea have recently attempted to provide for some degree of autonomy at a local school level, the reform documents are very influential, leading to directive, coherent, and rather uniform changes (Pang, 2000). This is especially true for elementary mathematics education, because Korea has only one kind of elementary mathematics textbooks and guidebooks authorized by the Ministry of Education and Human Resources Development (MEHRD). In other words, all Korean elementary school students study mathematics with the same textbooks, which consequently serves as bottom-line teaching and learning. Given this background, to understand Korean mathematics education we need to gain insight into these instructional materials. In particular, it is of great significance to explore how the principles and expectations concerning school mathematics manifest in the curriculum have influence on textbook development.
This paper introduces the general flows and directions in developing elementary mathematics textbooks and their related instructional materials, including teachers' guidebooks. This paper then presents main characteristics of the textbooks along with some representative cases. This paper finally deals with some issues on the future development of mathematics textbooks.

II. Overview of Development and Construction

1. General Flow

There are three kinds of Korean textbooks. First, the MEHRD consigns textbook development to an appropriate university or research institute. This is typical of most elementary school textbooks including mathematics textbooks, special education textbooks, and a few secondary school textbooks dealing with Korean language, history, and ethics. Second, multiple teams consisting of university-based educators and in-service teachers write the textbooks, which are assessed and certified by the MEHRD. Such case is typical of secondary school textbooks including mathematics textbooks. Third, non-educators write the textbooks, which are to be approved by the MEHRD. This is typical for some optional subject matter textbooks at the high school level such as computer education and religion.

Since this paper deals with elementary school mathematics textbooks, we explore the development process of the first kind of textbooks in detail (see Figure 1). The MEHRD announces a national mathematics curriculum and establishes an overall plan for developing instructional materials such as textbooks, workbooks, and teachers’ guidebooks. The plan is specific enough to provide textbook-writers with detailed guidelines. The MEHRD then selects a research and development institute and trusts the institute with the development of textbooks and their related resources. The assigned institute submits a proposal concerning the development, which is reviewed and finally approved by the MEHRD.

The institute organizes both a research and a writing team, constituting around 10 people per team. The main role the research team plays is to give a concrete blueprint to the national curriculum so that the basic principles of curriculum are specified in the textbooks. Given this, the institute determines the basic directions of writing textbooks and specifies what to do, which are reviewed again by the MEHRD. At this time, the MEHRD constructs a review and assessment board of textbooks in which many professors who specialize in mathematics education and experienced teachers are joined.
The writing team of the institute makes out manuscripts and goes through revisions as needed. On the basis of consultation with the research team, the first draft is prepared. The review and assessment board examines the draft and asks for revision. The institute revises the draft and the board reviews it again. The revised textbook is applied to scores of elementary schools for one year in order to diagnose the strengths and weaknesses of the textbook. The writing team prepares a second draft, drawing heavily on an analysis of school-based results, which is again consulted with the board. In this way, the final version of a textbook is made and published.
With regard to the general flow of developing textbooks, there are several aspects we need to note. First, as described above, the development of a new textbook and its related instructional resource is of great importance in terms of shaping Korean mathematics education, specifically at an elementary school level. Given this, we make every effort to develop a textbook that is as good enough to foster students' mathematical power as possible. Many contributing factors are involved such as a systematic and continuous review process up to developing the final version, a year-long experiment at a local elementary school level, discussion and consultation among experts with various backgrounds in elementary mathematics education, and so forth.

Second, since there is only one kind of elementary mathematics textbook, the MEHRD pays careful attention to select an appropriate research and development institute and to appoint a principal researcher who leads the overall process of textbook development. The research and the writing team of the institute have to consist of people who are teacher educators, practicing teachers with at least five years' teaching experience, and senior researchers with at least five years' research experience. As an effort to make writers fully responsible for the quality of textbook contents, each textbook includes the name of the author of each unit.

Third, the process of textbook development reflects the negotiation between practice and theory, or between university-based researchers and school-based teachers. On one hand, the textbook should be based on thorough research concerning the problems of current textbooks, the principles and directions of the updated national curriculum, the various perspectives of teachers, parents, and students, the accumulated results on how students learn mathematics, etc. On the other hand, the textbook should be customized in a way to reflect the needs and realities of elementary mathematics classrooms. For this reason, many experienced teachers are involved in writing textbooks.

However, teachers' involvement needs to be further explored in the international contexts. The professional leadership of mathematics teachers such as the National Council of Teachers of Mathematics (NCTM) has initiated the current reform movement in the U.S. and has made great efforts to change the culture of instructional practices. In particular, NCTM encourages mathematics teachers to fully engage in the process of the reform movement as directors of their own teaching practices and as partners with researchers or theorists. In Korea, the new curriculum, including updated instructional materials, has been implemented in a rather top-down format: Selected mathematics teachers are informed of the changes in curricular
emphases and the subsequent instructional implications, and then the teachers inform their colleagues. Some selected teachers are involved in making mathematics textbooks and workbooks. Although teachers’ involvement in developing textbooks has been encouraged and has indeed increased, the breadth of real engagement is rather minimal in the international contexts. Generally speaking, Korean mathematics educators develop a mathematics curriculum, textbooks, and guidebooks for teachers, and then teachers implement these well-developed materials.

2. Principles and Directions of Developing Textbooks

The basic principle in developing elementary mathematics textbooks\(^1\) is to follow and specify what the curriculum intends. The most recently developed seventh curriculum has a level-based differentiated structure and emphasizes students’ active learning activities in order to promote their mathematical power, which encompasses problem solving ability, reasoning ability, communication skills, connections, and dispositions. This curriculum resulted from the repeated reflection that previous curricula were rather skill-oriented and fragmentary in conjunction with the expository method of instruction, and that previous curricula did not consider various differences among individual students with regard to mathematical abilities, needs, and interests (Lew, 1999). The main motivations to the current curriculum include increasing concern for individual differences and the desire to provide maximum growth of individual students on the basis of their abilities and needs. Given the curriculum, mathematics textbooks intend to provide students with a lot of opportunities to nurture their own self-directed learning and to improve their creativity. To accomplish this purpose, several directions are established in developing elementary mathematics textbooks.

First, textbooks should consist of mathematical contents with which individual students can improve their own creative thinking and reasoning ability. At some point in a learning sequence, instructional resources are presented differently on the basis of individual differences of mathematical attainment. Whereas high-achieving students confront with advanced tasks including real-life complex situations, low-achieving counterparts solve basic problems involving the fundamental understanding of important mathematical concepts and principles.

\(^1\) Most Korean elementary mathematics textbooks developed under the previous curriculum have been translated into English and analyzed through Truman Faculty Research grants, Eisenhower Foundation funds and the National Science Foundation Award (Grow-Maienza, Beal, Randolph, 2003; see also http://eisenhowermathematics.truman.edu)
Second, textbooks should consist of mathematical contents which contribute to improving the process of teaching and learning. Most of all, textbooks have to underline a learning process by which students solve problems for themselves through individual exploration, small-group cooperation activities, or discussion.

Third, textbooks are to be easy, interesting, and convenient to follow on the part of students. For instance, instructions of games or activities in the textbooks should be specific enough for students to initiate them without a teacher’s further explanation and demonstration. Textbooks should take into account students’ various interest and stimulate their learning motivation. Textbooks should also consider various editing, design, and readability for students. Textbooks should also deliberate the appropriate use of different multimedia learning resources.

Fourth, textbooks are to be flexible in a way that teachers refine or even revise them reflecting on the characteristics of their schools or provinces. A textbook should be recognized not as the sole material to be followed but as an illustration of embodying the idea of the curriculum.

At this point, it may be informative to compare a recommended textbook with a traditional one in terms of the following six aspects (See Table 1).

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<th>Perspectives of Textbook</th>
<th>Traditional Textbook</th>
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<tr>
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<td>The sole &amp; the most significant textbook</td>
<td>Main but one among various instructional resources</td>
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<td></td>
<td>Mathematics education depends on the textbook</td>
<td>Mathematics education depends on curriculum</td>
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<td></td>
<td>Textbook mainly focuses on knowledge</td>
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<th>Statements in Textbook</th>
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<td></td>
<td>Summarizes knowledge, condenses concepts, lists the essential points for lecture</td>
<td>Presents various facts, provides specific cases, deliberates the process of learning (procedure &amp; method)</td>
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<td>The one and only structure applied to all textbooks</td>
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<th>Construction of Contents</th>
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<tr>
<td></td>
<td>Linear construction in terms of mathematical structure</td>
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<td></td>
<td>Monotonous construction of sentences and illustrations</td>
<td>Various designs &amp; editing</td>
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<th>Process of R &amp; D</th>
<th>Traditional Textbook</th>
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<td></td>
<td>Textbook development with minor basic research</td>
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First, a traditional textbook is regarded as the one and only, most important material teachers must follow and use in their teaching practices. From the traditional perspective on a textbook, school mathematics tends to depend exclusively on the textbook, which makes it difficult to accommodate the local needs and characteristics of various schools. A traditional textbook also centers around displaying mathematical knowledge in a systematic manner. In contrast, a recommended textbook is considered to be main but one among other different instructional materials. From this perspective, school mathematics relies on the curriculum rather than the textbook, which means that schools or teachers may feel free to change certain aspects of the textbook as long as it is in line with the basic principles or directions of the curriculum. A recommended textbook deliberates not solely on mathematical knowledge but also on skills and attitudes, and fosters creative thinking and reasoning ability.

Second, whereas a traditional textbook has a tendency to summarize knowledge, to condense concepts, and to list essential points for lessons, a recommended textbook presents various facts, provides exemplary cases, and considers the process of learning. In other words, a traditional textbook makes it easy for students to memorize many mathematical concepts and principles. In contrast, a recommended textbook makes it possible for students to understand the underlying mathematical structure beyond recalling simple concepts and principles.

Third, whereas a traditional textbook uses the same structure to develop units, its counterpart employs various methods so as to reflect on the different topics and characteristics of units. To be clear, a recommended textbook also considers consistency across units in terms of a general structure, but does not strictly adhere to such structure at the expense of the diversities coming from different content areas.

Fourth, knowledge and a teacher are the main factors in selecting the contents of a traditional textbook. Knowledge-based contents mean that we deliberate what to teach. Teacher-based contents mean that we consider who teaches. In contrast, the contents of a recommended textbook are determined by multiple factors beyond knowledge and a teacher; for example, real-life contexts related to fundamental concepts, concrete and diverse cases, students' experience and interest, utility of textbook contents, and the like.

Fifth, the contents of a traditional textbook are constructed linearly in terms of a mathematical structure. The sentences and illustrations of the textbook are rather monotonous and simple. In contrast, the contents of a recommended textbook are not constructed linearly; instead, they combine relevant knowledge and real-life experience. A recommended textbook is
also conscious of various editing and design in order to increase the effects of visualization on learning.

Finally, a recommended textbook is based on basic research more than a traditional one is. The basic research includes students' learning of mathematics, various needs of the current society, problems and issues of previous textbooks, strengths and weaknesses of educational technology, current trends in school mathematics, etc.

3. Tasting a Textbook: Structure of a Unit

The overall structure of constructing a textbook is based on the following guidelines:

- The contents of 6 mathematics areas\(^2\) may be distributed through a few units at each grade level.
- Units at each grade level should be balanced and unnecessary repetitions or illogical development should be avoided.
- The introduction of each unit should involve materials by which students are ready to learn mathematical contents in the unit with great motivation and interest.
- Appropriate examples should be included in a way that students are able to understand fundamental mathematical principles or structures.
- Various problems should be included so that students summarize and assess what they have learned.
- Each unit should embody the current curriculum with a level-based differentiated structure.

At each grade level, we have two mathematics textbooks for the spring and fall semester, respectively. A textbook consists of 8 units, each offering around 7 to 10 class sessions. For instance, units 3-1, which corresponds with the first semester for third graders, involve a) numbers up to 10000, b) addition and subtraction with three digit numbers, c) plane geometry, d) division, e) moving figures, f) multiplication, g) fraction, and h) length and time. Each unit includes about 3 to 7 learning themes. For instance, the unit on plane geometry has the following four themes: learning of angle, right triangle, rectangle, and square. Sometimes a few

class sessions are grouped together, presenting a block of activities with a single major theme. Teachers are expected to make decisions about how they divide the activities into given sessions on the basis of understanding the overall flow and sequence of the activities and understanding of their students.

**Unit Format:** The illustrated one-page opening of each unit, which appears in the first session, helps students figure out what they will study in the unit and get motivated to study what follows.

**Let's See in Everyday Life:** A word problem of a learning theme is introduced in real-life contexts. An expectation is that students may relate the learning theme to their daily life and may be motivated to learn.

**Activities:** Two or three activities are usually presented with concrete materials and thought-provoking questions. Students are expected to figure out basic mathematical concepts and/or principles by actively engaging in such activities. Whether or not students are able to define a mathematical concept on the basis of the activities depends on the learning theme.

**Let's Practice:** Students solve several problems related to the theme. A textbook offers practice in key concepts or principles students have covered in the class session. Usually simple and easy problems are introduced in the textbook, whereas various and difficult ones are presented in the workbook. Depending on the number of learning themes per unit, the above-mentioned three aspects are repeated per class session. In other words, a session of a 40 minute mathematics class usually starts with "Let's See in Everyday Life" and moves into "Activities" followed by "Let's Practice".

**Interesting Game:** After all the learning themes of the unit are covered, an "Interesting Game" is provided. This game is designed for performance assessment of the unit. While students are engaged in playing the game, teachers are expected to walk around and assess individual students' understanding of mathematical concepts or principles underlying the game.

**Problem Solving:** In this section, students have an opportunity to apply what they have learned to a new situation. This section usually covers the overall contents of the unit, not a specific learning theme.

**Unit Assessment:** Unit assessment consists of two parts. The first part is the same for all students as an overall examination of the unit, whereas the second is tailored to each individual. Specifically, "Let's See How Well I Have Learned " is presented for the purpose of evaluating
how much students have understood important mathematical concepts or principles of the unit. On the basis of the result of this examination, individual students may choose between "Let's Study Again" and "Let's Study More." The former is provided for low-achieving students in which fundamental concepts or principles in the unit are stressed and reinforced. The latter is addressed for high-achieving counterparts in which advanced thinking or complex applications are required.

4. Overview and Directions of Developing Workbooks

Each elementary mathematics textbook has its concomitant workbook. Whereas the textbook centers around mathematical activities and thinking processes by which students can learn mathematical concepts and/or principles, the workbook plays a major role in reinforcing such concepts and/or principles by letting them solve various problems. Textbooks are employed mainly in actual classroom instructions, but workbooks are usually for students’ self-practice after the school.

Since workbooks are used in tandem with textbooks, the following directions of developing workbooks are compatible with those of textbooks:

- Workbooks follow what the curriculum intends.
- Workbooks offer students to make use of mathematical knowledge and skills they learn from textbooks.
- Workbooks are to be easy to follow on the part of students for their self-directed learning and practice.
- Workbooks are tailored to individual students’ mathematical abilities.
- Workbooks cultivate students’ mathematical thinking, exploration, and problem solving ability.
- Workbooks consider their overall design in a way to increase the effects of visualization on learning.

Given the ideal of considering individual differences in the curriculum, workbooks have two kinds of problems, that is to say, basic and advanced problems. Basic problems are those that all students should be able to solve as long as they understand the underlying mathematical concepts and/or principles. Advanced problems are difficult ones that are mainly for the
students who easily solve the basic problems. These are designed to offer high-achieving students to face mathematical challenge and to extend their engagement with mathematics.

5. Overview and Directions of Developing Teachers' Guidebooks

As described, although there is no specific obligation to follow teachers' guidebooks, almost all Korean teachers use them with textbooks as their main instructional resources (Grow-Maienza, Beal, & Randolph, 2003; Kim, Kim, Lyou, Im, 1996). It is important to explore teachers' guidebooks in order to better understand Korean mathematics education.

The guidebooks intend to provide teachers with detailed explanations of the current curriculum, teaching and learning methods of mathematics, various instructional resources and their applications, theoretical backgrounds of given mathematical topics, and mathematical problem solving. Since the guidebooks, along with textbooks and workbooks, are developed under the same national curriculum, the following characteristics of developing guidebooks are compatible with those of other instructional materials:

- Guidebooks should reflect the directions and the main points of the current curriculum and are closely related to the contents of textbooks and workbooks.
- Guidebooks should describe helpful tips for instruction and assessment, in particular with regard to the topics of textbooks which may be difficult to implement at the elementary school level.
- Guidebooks should include problems for assessing students' prerequisite learning, eliciting their motivation to study, inducing their mathematical interest, and constituting a complimentary process for low-achieving students.
- Guidebooks should be specific enough for teachers to understand mathematical emphases and instructional methods of textbooks and workbooks.
- Guidebooks should include answers with exemplary solution processes of the problems in textbooks and workbooks.

**Unit Format:** Guidebooks consist of two parts. The first part deals with overall characteristics, directions, purposes, contents, and instructions of elementary mathematics education. The second includes exemplary lesson plans tailored to the main purposes of instructions such as concept-development, principle-exploration, problem solving and skill
automaticity. The second part then illustrates the following aspects of each unit of textbooks in detail.

**Classroom Episode:** A segment of reflecting the main characteristic of the unit is selected and illustrated in the form of interaction between the teacher and students in the classroom. It intends to help teachers be aware of the main points of the unit and the directions to teach at a glance.

**Overview of Unit:** This section summarizes the most important mathematical topics and processes students will encounter in the unit. It also emphasizes connections among mathematical contents, indicating a strong concern of explicit vertical linkages as well as horizontal connections for integrated thinking on the part of students. For instance, teachers are supposed to start with a diagnosis of students’ understandings in order to help them connect the current lesson with their previous knowledge structures. Implicit in this is the concern that students may learn mathematical knowledge as isolated so that they cannot retrieve together their knowledge related to solve problems in more complex or novel situations.

**Overall Plan of Unit:** This section provides a table with which teachers can see the learning theme, mathematical emphases, and main activities of each session. The table also indicates a specific page number of the workbook related to the session of the textbook so that teachers may guide students to develop their mathematical skills by solving the given problems.

**Explanation of Contents:** This section starts with rationales to teach the given unit and demonstrates how to use the illustrated one-page opening of the unit in a way to motivate students to learn. It then proceeds to detailed explanations and instructional procedures of the mathematical activities for each session, including how to set up and implement them, in line with background knowledge.

**Use of Workbooks:** This section replenishes a brief description of mathematical tasks of the related workbook, followed by their answers and exemplary solution processes.

**Supplementary Instructional Materials:** This section provides supplementary problems to be used for assessing students’ understanding. It also furnishes games and puzzles which can be used in the unit.

### III. Characteristics of Elementary Mathematics Textbooks

As described above, all Korean elementary schools use the same textbooks with concomitant resources such as workbooks and teachers’ guidebooks. Furthermore, these
instructional materials are the main resources for teachers to employ in their classrooms. As a result, a textbook is a strong determinant of what students have an opportunity to learn and what they do learn.

Main characteristics of elementary mathematics textbooks include relating mathematical concepts or principles to real-life contexts, encouraging students to participate in concrete mathematical activities, proposing key questions of stimulating mathematical reasoning or thinking, reflecting mathematical connections, emphasizing problem solving processes, assessing students' performance in a play or game format, and providing students with various problems for computational proficiency. Use of these characteristics is dominant, as can be confirmed by even a cursory examination of a textbook.

These characteristics for enriching learning environment for students are intended to support the curricular emphasis on the understanding of fundamental mathematical concepts or principles, logical thinking, problem solving, communication, and mathematical dispositions. Much of the current emphases in the textbooks reflects substantive shifts from learning as receiving to learning as understanding mathematical knowledge, and from emphasizing mainly problem solving skills and strategies to developing mathematical thinking and problem solving ability. In the following sections, I deal with each characteristic in detail with some background information and rationales. I also present examples so as to highlight key features and to better understand the characteristic as embodied in the textbooks.

1. Mathematical Concepts or Principles in Real-life Contexts

Through their experiences in everyday life, students gradually develop rather informal but complex and robust ideas about various mathematical topics such as numbers, patterns, shapes, data, etc. It has been emphasized that textbooks should intend to connect a rather formal mathematical knowledge in school to students' considerable knowledge base accumulated through everyday experience (e.g., Bransford, Brown, & Cocking, 1999; Kim, 2002).

Relating mathematical concepts or principles to real-life contexts makes mathematical knowledge meaningful and immediate for students. For many students, mathematics is merely another academic subject they ought to cover with considerable effort. In particular, for many Korean students, mathematics is often regarded as something abstract, irrelevant, and even useless for interpreting everyday phenomena and personal concerns (Cai, Lew, Morris, Moyer, Ng, Schimittau, 2004). They have a difficult time in connecting their knowledge of
mathematical concepts with understanding their daily life. What is learned in mathematics classrooms is considered something to be learned in school, but not something that is of practical significance for them. In fact, students apply mathematics mainly to the problems they solve in textbooks or tests, not to everyday events.

Against this background, current Korean textbooks intend to develop mathematical tasks that are related to students' everyday life, and to begin with concrete contexts before addressing rather abstract and formal knowledge. Students are expected to see that mathematical concepts are meaningful and they are indeed used in various instances of everyday life. This reflects the main purpose of Korean mathematics education that, "students should be able to understand basic mathematical concepts, principles, laws, and various relationships through mathematical exploration of different real-life phenomena or events" (Ministry of Education, 1998, p. 20).

Specifically, each mathematical topic is usually introduced in a textbook through the section "Let's See in Everyday Life." For instance, in Figure 2, a related mathematical topic is to find the common divisors of 8 and 12 (MEHRD, 2002, 5-1 textbook, p. 8). Students often have experience in facing with situations such as a birthday party or any kind of invitation party in which they have to allocate two kinds of materials evenly to a given number of dishes. The task encourages students to see the necessity of the concept of common divisors in concrete contexts where students have to divide 8 apples and 12 oranges evenly.

▶ Let's See in Everyday Life
You would like to put 8 apples and 12 oranges equally in each dish. Figure out how many dishes are needed.

[Figure 2] Introduction to Common Divisors

Another example is shown in Figure 3 (MEHRD, 2002, 6-1 textbook, p. 96). The task addresses the concept of proportion. Cooking is closely related to students' daily life and is also adequate for provoking their interest. Cooking calls for the exact measurement of given materials. Since a recipe provides the amount of materials per given specific number of people, a cook should usually be able to adjust the amount as needed. In this situation, the concept of proportion is naturally connected.

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3 This corresponds to the first semester for fifth graders.
2. Emphasis on Mathematical Activities

Despite the superior mathematics achievement by Korean students in international competitions, teacher-centered teaching practices have been considered a main problem in Korean mathematics education (Grow-Mainza, Hahn, & Joo, 1999; Pang, 2000). Korean teachers reported that their most common teaching method is to demonstrate how to solve given mathematical problems and then to provide students with similar problems for practice (Kim et al., 1996). Consequently, students are rarely actively engaged in developing their own solution methods individually, in small groups, or as a whole class. Learning mathematics is basically a repetition of receiving and practicing the teacher’s or textbook methods.

Against this background, current textbooks encourage students to actively participate in various mathematical activities. Instead of telling them how to do something, textbooks call for actually “doing” mathematical things. Textbooks intend to provide an intellectual challenge, stimulate discussion, and encourage cooperation by getting students to be immersed in an environment in which they themselves are actually performing mathematics rather than practicing rules. Textbooks use many active words such as conjecture, investigate, explore, pose, invent, justify, solve, explain, develop, represent, formulate, etc. This reflects the belief that it is only when students perform that they know mathematics.

Emphasis on mathematical activities is closely related to the use of concrete materials at an elementary school level. Physical materials encourage an exploratory approach and give
students something to think about or something to do. Furthermore, such materials can facilitate students to reflect on evolving ideas with the feedback coming from the materials. Textbooks include various materials embodying or illustrating the mathematical relationship that students are supposed to construct or understand.

In Figure 4, students are asked to figure out how to compute a typical subtraction problem of \(32 - 8\) with base-10-blocks (MEHRD, 2000, 2-1 textbook, p. 23). Before doing this activity, students are asked to solve the given problem with their own methods. The activity in Figure 4 intends to connect the concrete experience of manipulating base-10-blocks with a standard algorithm of subtraction with regrouping.

<table>
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<tr>
<th>Activity 2: Figure out how to compute (32 - 8) with base-10-blocks.</th>
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<tbody>
<tr>
<td>• Lay out 32 with base-10-blocks.</td>
</tr>
<tr>
<td>• Take away 8 from 12 units.</td>
</tr>
<tr>
<td>• What is (32 - 8)?</td>
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Method of Subtraction

![Subtraction with Base-10-blocks](image)

At the second-grade level, many students are able to subtract with pencil and paper but are often unable to explain, for example, the meaning of the little numbers that they write when they "borrow". It is also common for the students to fail to understand that the number after regrouping is the same quantity as before. They do not necessarily think about the concepts involved as they do the procedures. Roughly speaking, it is common for young students that conceptual knowledge and procedural knowledge of regrouping are disconnected from each other. Because of the lack of an understanding of why a procedure makes sense, students are often confused by the procedure itself and even tend to have a very negative view of what mathematics is all about.

The activity in Figure 4 first asks students to perform \(32 - 8\) with base-10-blocks step by step. The activity then calls for the answer and forces students to look back on the procedure and to think of why such an answer comes out. Students are expected to make sense of the
procedure with relation to the result. After that, the activity addresses the standard method of how to subtract. As a computational principle is formed and students learn how to record the procedure in a formal way, it is worthwhile to emphasize that the base-10-blocks that were used in the initial development now serve as a connecting link. In other words, the textbook intends to help students make connections between the principles and symbols mainly by linking the initial activity of using base-10-blocks to the procedure of standard subtraction algorithm.

Figure 5 shows another kind of mathematical activity in which students conjecture what happens when turning around a semicircular figure, do the experiment, confirm by drawing, and verbalize the resulting figure (MEHRD, 2002, 6-2 textbook, p. 28). Note that the activity includes both concrete activities and thinking processes. Instead of manipulating the given figure with an empty mind, the activity encourages students to imagine first and to talk about the resulting figure. Only then, does testing one's own conjecture with physical materials or simulations provide valuable feedback for understanding relationships.

**Activity 3**: Explore what kind of shape appears when a semicircular-shaped piece of paper is stuck to a chopstick and spun in a circle.

- First, imagine and talk about what kind of figure comes out.
- Spin the chopstick in a circle and explore what kind of figure appears.
- Draw a resulting figure on the right.
- What kind of solid figure is made?

[Figure 5] Rotation of a Semicircle

### 3. Emphasis on Mathematical Thinking

Mathematical thinking, in conjunction with problem solving, has been consistently emphasized as the most important part of students' mathematical experience throughout their school years. Since thinking mathematically is a conscious habit, it should be developed through consistent use in many contexts. Being able to reason is essential in making mathematics meaningful for students. In all content areas and at all grade levels, consequently, students need to develop ideas or arguments, make mathematical conjectures, and justify results.
The question of "why do you think so?" is the most salient feature in Korean elementary textbooks that are designed to elicit students' account of how they accomplish a given task. In fact, nearly all activities of textbooks include such a question at the end. Simply knowing the answer or solving a given problem itself is not enough. Thanks to questions such as "why do you think it is always true?", "why do you think so?", or "how do you know?", students come to realize that statements need to be supported or refuted by evidence, or something that is mathematically acceptable as an adequate argument. As students experience mathematical reasoning over and over, they come to know that mathematical reasoning is based on specific assumptions and rules.

We can find such an attempt to foster students' mathematical reasoning in a first grade textbook. It is based on the belief that even young students are able to express their thinking in their own words or using concrete examples. As they present their thinking to their groups and to the whole class, students can learn to articulate their reasoning by the norms of mathematical justification.

In Figure 6, students have to figure out how to handle with dividing a terminating decimal by a mixed fraction (MEHRD, 2002, 6-2 textbook, p. 76). The textbook shows that such a division can be approached both by rewriting the fraction as the decimal and by rewriting the decimal as the fraction. After computing in both ways, students have to determine which method is more convenient and explain why they think so. Simply completing the calculation or drawing the result is not regarded to be sufficient in this activity. Students are encouraged to compare the two approaches by the norm of mathematical convenience or easiness (Yackel & Cobb, 1996). Furthermore, as a related activity, students have to explore a similar division problem in two different contexts; one is where the quotient is a terminating decimal and the other is where the quotient is a non-terminating decimal. As in comparing the case of converting a decimal to a fraction with that of converting a fraction to a decimal in different contexts, students learn to evaluate their own thinking and develop mathematical reasoning skills by which they decide which calculation method is mathematically convenient or accurate in a given context.

**Activity 3: Figure out how to calculate** $0.36 \div \frac{1}{2}$

- Convert the fraction to a decimal and calculate (decimal)÷(decimal)
4. Emphasis on Mathematical Connections

One important characteristic of Korean textbooks has to do with internal coherence of mathematics. Stigler and Hiebert (1998) emphasize the importance of coherently designing both curriculum and lesson in a way that a central mathematical idea is carefully developed and extended. The notion that mathematical ideas are connected is permeated throughout the school mathematics experience at all levels. Korean textbooks intend to organize and integrate important mathematical ideas in a coherent way so that students can understand how the ideas connect with others and build on new knowledge and skills. Grow-Maienza, Beal, and Randolph (2003) found "the [Korean] textbook to be focused on the concepts in a concise, coherent, and systematic manner similar to what was observed in the classroom" (p. 1).

Emphasis on mathematical connections has to do with two rationales. First, when students can connect mathematical ideas, their understanding tends to be deeper and more lasting. Recent studies on students' mathematical learning have solidly established the crucial role of conceptual understanding (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, et al., 1997). When there are connections or integrations with existing ideas, we say that a mathematical concept is understood.

Second, mathematical connections help students understand the nature of mathematics. Very often students regard mathematics as a set of disconnected, isolated concepts and skills. Mathematical connections both within the curriculum of a particular grade and between grade levels assist students to develop a view of mathematics as a connected and integrated whole.

The following two examples reflect mathematical connections across grade levels and between topics within a grade, respectively. Figure 7 shows how the addition and subtraction of fractions with different denominators at a fifth grade level are based on other related concepts and operations at the previous grade levels.
Figure 7] Mathematical connections of addition and subtraction of fractions with different denominators
Figure 8 implies how the textbook can elicit mathematical connections in a class session (MEHRD, 4-2 textbook, p. 24). The given task is to compare 0.34 and 0.28. Students are first asked to represent each decimal on a grid paper and then determine the order. As emphasized above, students need to justify their ordering of the decimals. The activity also includes an important guideline, "Let's Think Again What We Have Learned", by which students are reminded of how they compared 0.5 and 0.8. Students need to be explicitly aware of the mathematical connections with regard to the ordering of decimals. This kind of guideline is evident throughout textbooks across all grade levels. Students develop a conscious habit to routinely ask themselves, "How is this problem or mathematical topic related to something I have learned before?". This approach requires them to be responsible for what they have studied and for using that knowledge to understand and make sense of new ideas. In this way, new concepts or principles are seen as extensions of previously learned mathematics.

<table>
<thead>
<tr>
<th>Let's See in Everyday Life: Hyojin and Suyeon ran. Hyojin ran 0.34km and Suyeon ran 0.28km. Who ran the furthest?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity 1: Order the decimals using a grid paper.</td>
</tr>
<tr>
<td>Color 0.34</td>
</tr>
<tr>
<td>Which is greater between 0.34 and 0.28?</td>
</tr>
<tr>
<td>Why do you think so?</td>
</tr>
<tr>
<td>Let's Think Again What We Have Learned</td>
</tr>
<tr>
<td>Talk about how to compare 0.5 and 0.8</td>
</tr>
<tr>
<td>0.5 has 5 of 0.1</td>
</tr>
</tbody>
</table>

[Figure 8] Ordering Decimals

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It has been well known that "all operations in Korean mathematics are systematically and coherently linked to the inverse relationships of addition and subtraction, and of multiplication and division" (Grow-Maienza, Beal, Randolph, 2003).
5. Fostering Problem Solving

Problem solving has been promoted as the centerpiece of school mathematics since the late 1980s in Korea. Practicing routine skills for optimal performance at the expense of understanding was seen as problematic in the 1980s. Instead of finding ways of achieving skill automaticity, the mathematics education community focused on students’ sense-making processes in solving mathematics problems. Practicing skills was left to problem solving situations that required application of skills.

In the past, the procedures or strategies of solving problems were emphasized. Students were often told the best way to solve specific types of problems. This approach has been criticized as something that shows only one path to the solution and does not promote mathematical thinking or problem solving ability as expected (e.g., Zambo & Hong, 1996). In recent years, problem solving has been regarded not only as a goal of learning mathematics but also as a major tool of doing so. Problem solving is an integral part of mathematics learning across different content areas at all grade levels. In fact, every unit of textbooks has a special section of problem solving at the end and each grade has one or two whole units dealing with how to solve problems.

Frequently cited strategies in the elementary mathematics textbooks include drawing diagrams, carrying out an experiment, establishing an equation, making a table, working backwards, looking for patterns, guessing and checking, creating a simpler problem, and so forth. Explicit instructional attention is drawn on these individual strategies through the 1st grade to 4th grade. At upper grade levels, students are expected to learn how to express, categorize, and compare their strategies. They should recognize when various strategies are appropriate to use and should be capable of deciding when and how to use them.

Figure 9 shows a typical case in which students have to solve a problem using different strategies (MEHRD, 2002, 6-1 textbook, p. 122). Solving the problem itself is not the focus. Instead, the attention is given to employ different problem solving strategies and compare them each other. In this way, textbooks help students develop the ability to decide which one to use, and be able to adapt and even invent strategies on their own.

Another aspect that needs to be addressed has to do with problem posing. In comparison with problem solving, problem posing is rather new in mathematics textbooks. Given the research that posing problems comes naturally to even young students (e.g., Brown & Walter, 1990), textbooks came to include posing problems in various contexts.
Solve a Problem in Different Ways

Let's See in Everyday Life

Jihae spent 4000 won to buy a total of 23 erasers and pencils. The price per eraser is 200 won and the price per pencil is 150 won. How many erasers and pencils did she buy?

Activity 1: Solve the problem with guessing & checking

- What is looking for?
- Guess the number of erasers and pencils.
- Check whether or not the total price of erasers and pencils is 4000 won.
- If the price is not 4000 won, which should be increased?
- Repeat this process until the price comes to be 4000 won.
- How many erasers and pencils are there when the price is 4000 won?

Activity 2: Figure out the number of erasers and pencils by making a table.

- What are we trying to find out?
- What are the givens?
- Make a table including the number of erasers and pencils, and the price.

<table>
<thead>
<tr>
<th># of erasers</th>
<th># of pencils</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Fill out the table in a way that the sum of the numbers of erasers and pencils is 23.
- Compute the price, respectively.
- How many erasers and pencils are there when the price is 4000 won?
- Explain the solution process.

[Figure 9] Solving a Problem in Different Ways

Figure 10 shows a task that requires students to pose a problem using the given information and to solve it (MEHRD, 2001, 3-1 textbook, p. 84). Because students are not familiar with posing problems, the textbook includes an illustrative example. As students are encouraged to explore, share failures and successes, and question on another, they are more likely to pose problems and to persist with challenging problems.

6. Performance Assessment in a Play Format

Another characteristic of Korean elementary mathematics textbooks has to do with performance assessment. Previous textbooks tend to focus on formal assessment to judge what individual students can do mainly on paper-and-pencil tasks, with limited time to complete
Posing Problems

Read the following paragraph, pose a problem, and solve it

Jaehee bought a double-colored paper with 14 groups of 6 each and a single-colored paper with 12 groups of 8 each.

Example:

How many sheets of double-colored paper did she buy?

14 X 6 = 84

Jaehee bought 84 double-colored paper.

Hyungmin's school collected books to send them to her affiliated school.

There were 4 boxes of 32 storybooks each and 3 boxes of 18 biographies.

[Figure 10] Posing and Solving Problems

given tasks. However, current textbooks consider other kinds of assessment that support the learning of important mathematics and furnish useful information to both teachers and students. This is based on the claim that assessment should help teachers make instructional decisions and that assessment should allow each student to demonstrate his or her best strengths in different ways (NCTM, 2000).

Given that assessment should be an integral part of instruction, each unit of textbooks includes an "Interesting Game" in which students in pairs or in small groups play a game that is related to the important mathematical contents of the unit. Students are usually required to apply what they have learned to complex or new game situations. Meanwhile, teachers are expected to assess individual students' mathematical progress through informal means such as careful observations or questions and to provide any necessary prompts. In this way, assessment becomes a routine part of the ongoing classroom activity rather than a test at the end of instruction to see how students perform under special conditions.

Figure 11 is a game in the unit of multiples and divisors (MEHRD, 2002, 5-1 textbook, p. 16). Students are asked to place as many as Paduk stones as shown on the number card in a rectangular figure. For example, if there is the number 6 on the card, students are expected to put stones in the forms of 1x6, 2x3, 3x2, 6x1, respectively. This game is about finding the divisors of a given number. While students play games in pairs, teachers have to observe and determine whether they understand divisors and are able to apply the concept to the game situation.
Interesting Game
Materials: Board, Paduk Stones (White, Black), Number Card(6, 8, 10, 12, 16)

Play: Play in pairs ① Mix the number cards and place them face down. ② A player flips one card over. ③ Place as many Paduk stones as shown on the card to form a rectangular figure in the board. ④ Repeat ③ so as to make different rectangular figures within a limited time. The winner is whoever has more figures. (If the number of figures is the same, the winner is whoever makes first.)

* If you are the winner, mark 0 in the table.

<table>
<thead>
<tr>
<th>Turn</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td></td>
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</tbody>
</table>

[Figure 11] Interesting Game: Figuring out Divisors

Figure 12 is a game about basic multiplication facts (MEHRD, 2000, 2-2 textbook, p. 23), and is in fact very popular even to adults. This game is easy to follow and is not restricted to the number of people or places. After learning the meaning of multiplication, students are encouraged to memorize a times table from 2x1 to 9x9. The game provides students with a great opportunity to recall basic multiplication facts in an enjoyable format.

Interesting Game

Play
• Play with several people sitting in a circle.
• Clap your hands and recite, "Let's memorize multiplication facts!"
• A player poses such problems as '2x3=?', '7x5=?' and the next player answers.
• If the answer is wrong, the next person answers.
• Clap your hands and recite, "Let's memorize multiplication facts!"
• Repeat this process.

[Figure 12] Interesting Game: Memorizing Multiplication Facts
7. Mastering Mathematical Skills

Developing computational proficiency is another characteristic of Korean elementary mathematics textbooks. The textbooks have consistently emphasized the importance of a balance and connection between conceptual understanding and computational proficiency. On one hand, over-practiced computational methods without understanding have been often criticized. On the other hand, it is noted that understanding without computational fluency can inhibit the problem solving progress.

In recent international comparisons of mathematics achievement, East Asian students have consistently demonstrated their superior performance not merely in skill-oriented problems but also in concept-oriented ones that require complex thinking and mathematical reasoning (e.g., Mullis et al., 1997, 2000; OECD, 2001). As explaining what accounts for such a superior achievement, Leung (2002) claims that repeated practice in a systematically well-designed curriculum may become an important way to understanding. As he put it,

The process of learning very often starts with gaining competence in the procedure, then [students begin to understand the concepts behind the procedures] through repeated practice. This is especially the case for elementary school children. ... Very often, elementary school children need to practice without [obtaining a] thorough understanding first, and through practicing …[such] procedures they progressively [come to] understand the concepts (p. 13).

Korean elementary mathematics textbooks include the section of "Let's Practice" in almost all class sessions by which students have consistent opportunities to review key concepts or principles they have learned. Various kinds and levels of problems in workbooks also help students become confident and fluent in their arithmetic computation. While workbooks are used in spare time, and not usually during regular mathematics class, a teacher's appropriate guidance and check-up helps students to be responsible for their practice.

IV. Conclusion

Due to the importance of elementary mathematics textbooks in the Korean context, we have consistently made great efforts to develop good ones. Although the principal characteristics of current textbooks are desirable in many respects as described above, there is
always room for improvement. This section deals with future directions with regard to the development and the construction of instructional resources.

1. Directions for Developing Textbooks

First, it is open for discussion that elementary school textbooks should be developed by multiple teams and later certified by the MEHRD, rather than by the university or the research institute consigned by the MEHRD. In other words, elementary mathematics textbooks may need to be developed in the same way of secondary counterparts. On one hand, this kind of development method may lead to diverse and creative textbooks, partly because free competition among many experts is allowed. On the other hand, this method requires more financial support. It may be a practical issue in that the funds for research and development of textbooks and the number of researchers are poor in comparison with those of the United States., United Kingdom, German, France, and Japan (Lee et al., 1995). Multiple textbooks also require well-grounded criteria by which we differentiate good resources from bad ones.

Second, a field test of textbooks needs to be emphasized and scrutinized. Beyond mere teacher interviews and surveys, classroom observations and subsequent analyses should be made in detail. Such methods can urge both the writing team and the review team of textbooks to be sensitive about what works in the actual classroom situation and what does not, so that they develop better textbooks grounded in classroom contexts.

2. Directions for Constructing Instructional Materials

Since the new curriculum and concomitant instructional materials have been sequentially operated in schools only from 2000, the outcomes are yet to be measured. Some issues to be considered, however, are raised from case studies on applying the current textbooks to elementary school classrooms (e.g., Pang, 2002). Such issues are related to the emphasis on activities without mathematical essence, the use of concrete manipulative materials or technology without connecting the underlying concepts or principles, the overuse of worksheets, and so forth.

The followings are several aspects we have to deliberate on in revising the current instructional materials. First, connections among mathematical topics and other contexts need to be emphasized. To be sure, current textbooks and teachers’ guidebooks consider the mathematics that has been studied by students at the previous level and what is to be the focus
at successive levels. Nevertheless, they are lack of the tasks in which students are urged to piece multiple mathematical concepts or principles together. Besides, although current materials underline mathematical topics as embedded in real-life situations, it is questionable whether such situations are "authentic" in a way that students really face to in their everyday life. Learning mathematics involves not solely accumulating ideas but building successively more refined understanding. Well-connected instructional resources are necessary to guide students to increasing levels of sophistication and depths of knowledge.

Second, textbooks need to consider seriously how to promote students' mathematical dispositions. Students, even with good achievement, develop increasingly negative mathematical dispositions and feel lack of self-esteem with regard to their mathematical ability (e.g., Kim et al., 1996; Sorensen, 1994). In fact, the TIMSS and TIMSS-R found that the countries with the highest performance in mathematics including Korea also had students developed the most negative perceptions of mathematics and success in the subject (Mullis et al., 1997, 2000). Current textbooks include more concrete activities and plays as a part of attempts to elicit students' participation of and interest in mathematics than did previous textbooks. However, many elementary school teachers think that it is not enough (Lee, 2001). It is high time for textbook-writers to ponder carefully on whether the activities of textbooks are appropriate to provoke students' interest, whether they are easy to implement in any classroom situation, whether they are suitable to accomplish within a 40 minutes classroom session, and so forth.

Third, given that current textbooks focus mainly on number and operations, in particular four basic operations, they need to consider the balance among various mathematical topics. Comparative studies of elementary mathematics textbooks between Korea and other countries show that Korean textbooks are poor in dealing with basic topological concepts such as inside/outside, open/closed, between and order, various solid figures from the early grades, different geometric patterns, and multiple methods of making a table or a graph (Kim, 1999).

Fourth, Korean textbooks need to include more concrete tasks which nurture students' problem solving, reasoning, communication, and representation ability (Pang, 2002). Although the national curriculum emphasizes the significance of mathematics as a process with regard to fostering students' mathematical power, textbooks are not specific enough for teachers to guide what they have to do in their actual classroom situations. For instance, many elementary school teachers recognize the importance of communication in mathematics classrooms, but they do
not know how to increase students' communication skills and orchestrate their discourse. If mathematics as a process is important, textbooks and teachers' guidebooks should embody how to foster problem solving, reasoning, communication, and representation in detail. For example, if students need to develop communication ability in mathematics, they should have a lot of opportunities to verbalize mathematical concepts, to explain and justify their solution strategies, to pose problems, and to evaluate other students' strategies or explanations. Such opportunities require a well-articulated curriculum so that teachers understand what they have to do in a given class session.

Finally, teachers' guidebooks need to include more detailed explanations of curriculum and various instructional materials which teachers are ready to implement in the classroom. A preliminary survey shows that teachers are less satisfied with guidebooks than textbooks (e.g., Lee, 2001). Given that the teacher is the person who makes the curriculum come alive in the classroom, the guidebooks should offer materials to support his or her own professional development. Inherent to this is that the guidebooks should include the best information and richest materials such as different activities for a wide variety of situations, detailed illustrations of how to implement them, and articulated guidelines of how to assess students. Ultimately, the teacher uses the guidebooks in ways that make sense for his or her peculiar teaching style, the particular group of students, and the constraints and supports of particular school contexts.

References


