TSG 15: The role and use of technology in the teaching and learning of mathematics

Team Chairs

Lulu Healy, Centre of Exact and Technlogical Sciences, Pontifical Catholic University of S“o Paulo

Address: Rua MarquÍs de Paranagu· 111, ConcolaÁ“o, S“o Paulo SP 01303-050, Brazil

lulu@pucsp.br

Jim Kaput

Department of Mathematics, University of Massachusetts - Dartmouth

Address: 285 Old Westport Rd., No. Dartmouth, MA 02747-2300,

jkaput@umassd.edu

Team Members

Rosihan M. Ali, School of Mathematical Sciences, Malaysia University of Science, Malaysia

rosihan@cs.usm.my

Andrej Blejec, National Institute of Biology, Slovenia

andrej.blejec@uni-lj.si

Luc Trouche, IREM, University of Montpellier II, C.C. 040, France

trouche@darboux.math.univ-montp2.fr

Aims and Focus

We hope that the ICME 10 Topic Study Group The Role and Use of Technology in the Teaching and Learning of Mathematics will serve a dual role: as a forum in which mathematics educators may come together to discuss and to probe major issues associated with the integration of technology into scenarios associated with mathematics teaching and learning and as a place where we can share our ongoing work and perspectives.

With this in mind, our aim is to organise the Topic Group so that we can take advantage of the special event of ICME to do two things:

(1) To step back and think about the overarching big ideas and long term trends related to technology in mathematics education, with a focus on promising directions and foundational issues.

(2) To share participantsí ongoing work and perspectives in ways that inform the longer view of technology in education and vice-versa, to be informed by the longer view.

To serve these ambitious goals, we propose to organise the topic group around a set of three inter-related themes as well as to utilise web-presentations in advance of ICME 10. The themes proposed are:

(1) Mathematical Thinking, Technology and the Evolution of Mathematics - Clarifying the Reciprocal Relationships
This theme is suggested in order to bring together those interested in theoretical models of the ways in which technology shapes, and is shaped by, learners' mathematical activities and developing meanings. Additionally, it will provide the place to discuss the co-evolution of Mathematics and Technology and examine the new mathematics that particular technologies make possible, both from a historical perspective (looking back) and a with an eye to future developments (looking forward).

(2) Orchestration of Mathematics Teaching in the Presence of Technology - Understanding Structure in the Variation
The focus in this theme will be on pedagogic mediation of technology-integrated learning scenarios and how different instructional approaches and classroom organisations impact mathematical interactions in classrooms. Issues connected to the design and configuration of technology, tools and tasks for technology-rich learning scenarios ñ and in particular the principles that seem to be at work ñ will also be given space in this theme.

(3) What Are Key Factors in the Design of, and What Are Implications of such New Technologies As Classroom Networks, New Actions, New Representations, and New Devices?
This future-oriented theme intends reflection on the current and rapid future development of new classroom technologies (software, wireless networks, and new device-types) and their potential to yield new forms of teaching, learning and participation. For example, what new options are open when students and teacher can send mathematical objects fluently across diverse devices in a wirelessly connected classroom? How are assumptions about the learnability of topics challenged by new technology capacities?

Our objective in choosing this strongly thematic approach is to advance the field by providing a coherent, forward-looking view that helps people think ahead in productive ways. It is also the case that our Topic is extremely broad, including all age level students, all mathematical topics, and all kinds of technology, so establishing coherence is a major challenge. We hope that our work will also help lay the base for published products, including summary papers, books or monographs.

Call for Papers

We welcome contributions explicitly related to the three themes outlined above. Because of the limited amount of time allocated to the topic study groups and an expectation of several dozen papers, we have decided to host paper presentations in poster sessions (see programme). While no page limit is in force, we strongly prefer shorter papers ahead of longer ones, particularly for inclusion on the web site. All the accepted contributions will be made available on the Topic Group web-site prior to the Congress and all contributors invited to present their work in poster form during the second session, organized by theme. During this session, it is expected that the contributors will be available to discuss their work with the other group members,. The intention is hence to permit a form of oral presentation of all accepted work. Contributors will also be invited to bring copies of accepted papers, including expanded versions, and CDís to be presented-by-distribution during this session.

All proposals will be reviewed by members of the organising group. This review procedure will lead to three possible outcomes: (1) acceptance for poster presentation and distribution, (2) recommendation for revision, or (3) rejection.

Proposals will be accepted until February 15 and information about the acceptance of papers will be available by March 15. Note that earlier submissions allow for the possibility of revision.

Practical Information

Papers should be a MAXIMUM of 8 pages in length. They should be written clearly in English. Text should be 14 point TIMES with 16 point spacing and should fit into an outline of 16 cm x 25 cm. Papers should start with an abstract of up to 10 lines, single spaced and indented 1cm from the left text edge. Spacing between paragraphs should be 12 points. The title should be in 16 point bold capitals, followed by authors' names and institutions in 14 point italics, all centred in the text; name(s) of participating authors should be underlined.

Papers in .doc or .pdf format should be sent to both Topic Group chairs.

Programme

The topic study group has been allocated three one hour sessions and one ninety minute (final) session. We intend to set up a web-site so that web-based presentations of participants' work is available in the weeks prior to ICME 10. Our goal is that this Topic Group will lead to published products following the Congress.

Session 0: Web-based preliminary presentations by invitees and presenters in the weeks prior to ICME 10.

Session 1: Three summary plenary 20-minute theme-based presentations by invited speakers addressing one theme and referencing the accepted papers. These will be made Web-available in advance of ICME. Participants select a favorite theme and meet informally to discuss it after the session led by that themeís invited speaker.

Session 2: A dual poster session. One will be for paper-based posters including papers-for-distribution (where desired) organized according to the three themes and further grouped by student age-level. The other will consist of repeating 15 minute parallel live demonstrations of new technologies addressing the issues of Theme 3.

Session 3: An invited 15 minute reaction to the three plenary presentations followed by two parallel theme-based discussion sessions. These discussions, based on the first two themes, should yield plans for the session 4 report, which may include ideas for a publishable product.

Session 4: Each theme-group gives a 10 minute plenary presentation focused on promising directions and foundational issues reflecting their discussions during previous sessions. This will be followed by a 20 minute invited reaction identifying cross-cutting issues and summary. Then the entire TSG plans future activity - perhaps a book, monograph and/or synthesis papers, and perhaps a continuing web site.

Papers and Discussion Documents

Plenary Papers

AUTHOR(S)	TITLE	PDF FILE SIZE

Jim Kaput	Technology Becoming Infrastructural in Mathematics Education	Download file (112 KB)
Bibi Lins (Abigail Fregni Lins)	Dynamic Geometry Software: Two Ways of Seeing It and Using It	Download file (64 KB)
Luis Moreno Armella	Mathematical Thinking and Technology: Some Views on Their Co - Evolution	Download file (124 KB)
Richard Noss Ceclia Hoyles	The Technological Presence: Shaping and Shaped by Learners	Download file (72 KB)

Accepted Papers

AUTHOR(S)	TITLE	PDF FILE SIZE
Sergei Abramovich Peter Brouwer	Hidden Curriculum as a Didactical Framework for Mathematics Teacher Education in a Technological Paradigm	Download file (108 KB)
Maxima J. Acelajado	The Impact if Using Technology on Students í Achievement, Attitude, and Anxiety in Mathematics	Download file (200 KB)
Rosihan M. Ali Kor Liew Kee	Undergraduate Mathematics Enhanced with Graphing Technology	Download file (220 KB)
Abel MartÌn Álvarez Jordi Baldrich Álvarez	Statistics and the New Graphing Calculators	Download file (92 KB)
Mette Andresen	Flexibility of Mathematical Conceptions: Identification, Role and Causes	Download file (168 KB)
Marcelo Bairral Leonardo Zanette	Learning and Teaching Geometry in Virtual Environments	Download file (60 KB)
Gail Burrill	Teaching and Learning Mathematics Using Handheld Graphing Technology	Download file (104 KB)
Sangsook Choi - Koh Dr. Hokyoung KO	Prospective Math Teachers í Thinking in Explorations Using a Hand - held Calculator	Download file (112 KB)
Alison Clark - Jeavons	Integrating the Use if ICT in the Mathematics Classroom: Developing Teachers í Practices	Download file (100 KB)
Tilak de Alwis	A Center of Gravity Approach to Conic Sections and Dynamic Geometry Software	Download file (1.2 MB)
Ziva Deutsch Akiva Kadari Thierry Dana - Picard	ì Ezer - Reshetî — Distance Mentoring	Download file (132 KB)
David Driver	Problem Solving in a CAS Environment	Download file (364 KB)
Kamon Ekthaicharern	A Study of Undergraduate Mathematics Students í Achievement and Attitudes in Learning Linear Algebra by Using Graphing Calculators	Download file (120 KB)
Eleonora Faggiano Luciano Faggiano	Math Cooperative Learning Networking Technologies	Download file (112 KB)
Patricia Forster	Teaching and Learning Statistics with Technology	Download file (168 KB)
Claudio Fuentealba Ruth Galindo	Function Sequence Using Software Support	Download file (56 KB)
Victor Giraldo Luiz Mariano Carvalho	Computational Descriptions and Conflicts Associated with Irrational Numbers	Download file (92 KB)
Derek Glover Dave Miller Doug Averis	Panacea or Prop: The Role of the Interactive Whiteboard in Improving Teaching Effectiveness	Download file (232 KB)

Luiz Carlos Guimarães Rafael Barbastefano Franck Bellemain Elizabeth Belfort	Tools for Distance Teaching in Mathematics	Download file (184 KB)
Hulya Gur	Calculator Supported Hand - Outs for Teachers to Solve Real World Problems	Download file (56 KB)
Lenni Haapasalo Djordje Kadijevich	Using Innovative Technology for Revitalizing Formal and Informal Mathematics: A Special View on the Interplay Between Procedural and Conceptual Knowledge	Download file (908 KB)
Alena Hospesov á Helena Binterov á	Investigations in Excel - Aided Mathematical Learning	Download file (2.1 MB)
Rosalyn Hyde	A Snapshot of Practice: Views of Teachers on the Use and Impact of Technology in Secondary Mathematics Classrooms	Download file (104 KB)
J. Imai Y. Okouchi D. Watabe H. Komatsugawa	Development and Case Studies of a WBT System for Remedial Mathematics	Download file (68 KB)
Fabrizio Iozzi Guido Osimo	The Virtual Classroom in Blended Learning Mathematics Undergraduate Courses	Download file (148 KB)
Ho Kyoung Ko	The Role of the Graphing Calculator Through Characteristics of Verbal Interaction: A Case Study	Download file (196 KB)
Joong Kwoen Lee	A Case Study of Preservice Teachers í Learning Mathematics Under the Computer Technology Environments	Download file (96 KB)
Shuk - kwan Susan Leung	Using Asynchronous Internet - Based Technology: Case of Graduate Course in Mathematical Problem Solving	Download file (244 KB)
Bibi Lins (Abigail Fregni Lins)	Towards New Trends on the Role of Users of Technology: A Look at Some Research Fields	Download file (60 KB)
Malgorzata Makiewicz	Role of Photography in Developing Matheamtical Creativity in Students at Elementary and Practical Levels	Download file (1.6 MB)
Hideaki Miyashita	Method of e - Means/Infrastructure for Information Design Oriented Quality Training of Mathematics Teacher	Download file (648 KB)
Ambj – rn Naeve Mikael Nilsson	ICT Enhanced Mathematics Education in the Framework of a Knowledge Manifold	Download file (1.5 MB)
Masahiro Nagai Katsuya Shiraki Hiroaki Koshikawa Kanji Akahori	Mathematics Problem Solving Using a Web - Based Knowledge Map and Analysis of the Process	Download file (1.2 MB)
Mikael Nilsson Ambj – rn Naeve	On Designing a Global Infrastructure for Content Sharing in Mathematics Education	Download file (520 KB)
Federica Olivero	Hiding and Showing Construction Elements in Cabri: A Focusing Process	Download file (52 KB)
Elzbieta Perzycka	Computer - Aided of Maturity to Learning the Mathematics in School Conditions	Download file (40 KB)
Jennifer Piggott	Developing a Framework for Mathematical Enrichment	Download file (48 KB)
Peter Ransom	Estimating Angle: Working with Real Data in the Classroom	Download file (712 KB)
Research Group of Integration of Mathematics Curriculum and Teaching Materials in High School with Information Technology (Jianyue Zhang)	Research and Practices of Integration of Mathematics Curriculum and Teaching Materials in High School with Information Technology	Download file (84 KB)
Cristina Sabena	The Transparency of Instruments as Index of Perceptive and Cultural Relation to Concepts	Download file (157 KB)
Heinz Schumann	New Methods from New Facilities in DGS: The Case of Algebraic Curves	Download file (184 KB)

Gabriel J. Stylianides Andreas J. Stylianides	Reconsidering the Drag Test as Criterion of Validation for Solutions of Construction Problems in Dynamic Geometry Environments	Download file (132 KB)
David Thomas Cynthia Thomas	Lessons Learned: Fostering a Pedagogic Frame of Reference for the Use of Mathematical Modeling Technologies	Download file (144 KB)
Jos Tolboom	Working with Completely Digital Materials in Secondary Mathematics Education	Download file (100 KB)
Jerry Uhl	Using Technology in Math Before Calculus	Download file (40 KB)
Marja - Leena Viljanen	Computer - Based Learning Environment for Secondary School Algebra	Download file (248 KB)
Jenni Way	Multimedia Learning Objects in Mathematics Education	Download file (404 KB)
Hans - Georg Weigand	Mathematics Teacher Education on the WEB (MaDIN) — An Internet - Supported Teaching and Learning System	Download file (320 KB)
Zhang Xiong	Mathematical Education as Social Structure	Download file (56 KB)
Zhang Xiong	Trend of Mathematical Education Reform in China	Download file (44 KB)
Bernhard Zgraggen	Interactive, Generic, Heuristic and Dynamic Step - by - Step Solutions to Mathematical Problems in the World Wide Web	Download file (200 KB)

A STUDY OF UNDERGRADUATE MATHEMATICS STUDENTS' ACHIEVEMENT AND ATTITUDES IN LEARNING LINEAR ALGEBRA BY USING GRAPHING CALCULATORS

KAMON EKTHAICHARERN
SRINAKHARINWIROT UNIVERSITY
BANGKOK, THAILAND

kamon@swu.ac.th

ABSTRACT

This study attempted to explore the possibility of using graphing calculators at the undergraduate level by focusing on Linear Algebra.
In particular, it proposed to study the effects of graphing calculators on instruction and the classroom social norm, students' achievement and attitudes in learning Linear Algebra by using graphing calculators.

Twenty-eight undergraduate mathematics students participated in the study. Each of them was given a TI-92 graphing calculator to be used while studying Linear Algebra as well as in taking the tests.

It was found that the teaching and learning covered more materials and more deeply. Time was available for focusing on understanding and solving various problems. Students had a positive attitude toward the use of graphing calculators. Students indicated that graphing calculators were easy to use. They saved time in calculating and made the students feel confident. Students also thought that this was a good experience for them and suggested the use of graphing calculators with other groups of students. The graphing calculator classroom atmosphere was a good learning atmosphere and produced a lot of discussion without tension.

Scores on the achievement test indicated that students were doing well. The achievement difference (the difference between scores in the Principle of Mathematics, a pre-requisite to Linear Algebra, and Linear Algebra) for each student in the low achievement group and high achievement group was calculated for analysis.

Introduction
Learning and teaching mathematics nowadays should emphasize understanding, problem solving, reasoning, analyzing and applications. In order to achieve these purposes, teachers and students should teach and learn with media and technologies, such as computers and graphing calculators, to make abstract things visible as objects, to animate stable pictures and to compute basic mathematical calculations which are considered to be unnecessary for students. The graphing calculator could be used to save time so that students could learn and

understand more topics, solve various deep problems and solve real-world problems, which can hardly be done without technology. Using this technology, the students could observe the results and make conjectures for themselves. This method would help the students understand and get new knowledge by themselves, according to the focus-on-learner teaching process.

Waits & Demana (Waits & Demana, 2000) have found that calculators cause changes in the way we teach and in the way students learn. Before computers and calculators, it was necessary for students to spend time mastering and becoming proficient in the use of paper-and-pencil computational and manipulative techniques. Today much of this time can be spent on developing deeper conceptual understanding and valuable critical-thinking and problem-solving skills.

Dick (Dick, 1992) and other research works, such as McClendon (McClendon, 1992), Runde (Runde,1997), Siskind (Siskind, 1995) and Wilkins (Wilkins,1995), indicated that using calculators for learning mathematics could help students in problem solving so that they will have more learning time, not worry about computations and instead focus on the method of problem solving.

Moreover**,** Hollar (Hollar, 1997), Kinney (Kinney, 1997), Browning (Browning, 1989) and Slavit (Slavit, 1994) indicated that graphing calculators could help in developing deeper conceptual understanding because graphing calculators could draw pictures to help the student to understand better about graphs and functions and to integrate the several fields of mathematics, such as graphs, numbers and algebra.

Purposes of the study

1. To study the achievement of undergraduate mathematics students who learn Linear Algebra using graphing calculators.
2. To compare the achievement difference between a high achievement group and a high achievement group.
3. To study the attitudes of the students in the sample who learn Linear Algebra using graphing calculators.
4. To study the learning, teaching and social norm of a class that learns Linear Algebra using graphing calculators.

Hypotheses of the study

Hypothesis 1: More than 60% of the students who learned Linear Algebra using graphing calculators would pass the criteria.

Hypothesis 2: The achievement difference between the high achievement group and the low achievement group would be significantly different.

Hypothesis 3: The attitudes of students in the sample who learn Linear Algebra using graphing calculators would be at the good level.

Design of the experiment

Sample group

The sample group for this experiment was 28 junior undergraduate students, mathematics major, Faculty of Science, Srinakharinwirot University, Thailand, who registered for the Linear Algebra course in the 1st semester 2002 academic year. The duration of the teaching experiment was 15 weeks, 3 hours per week.

High achievement group and Low achievement group
The classification of the sample group into a high achievement group and a low achievement group was based on the score of basic knowledge in mathematics of the students, derived from the Principles of Mathematics course (MA 241). This course is a pre-requisite course of Linear Algebra and the students in the sample group were taught this course by the same instructor. The group of students in the sample group who got scores of at least 60 % on MA 241 was called the high achievement group, and the ones who got scores of not more than 50 % was called the low achievement group.

Experimental tools
The experimental tools consisted of a TI-92 graphing calculator, Linear Algebra text book, worksheets composed by the researcher, achievement tests and a questionnaire about the attitudes of students in learning Linear Algebra using graphing calculators. Each student had a TI-92, a Linear Algebra text book and worksheets.

Data collection in the experiment
In the experiment data was collected as follows:

Achievement in learning Linear Algebra was based on the score from achievement tests and from 15 Home Quiz worksheets, totaling 100 marks. The criterion for an achievement pass in learning was a score of at least 60 % of the total marks.
Achievement difference was computed by taking the achievement score in learning Linear Algebra minus the achievement score in learning Principles of Mathematics.

Attitudes of Students who learned Linear Algebra using graphing calculators came from the data obtained from the questionnaire. There were two parts in the questionnaire. The first part contained 15 statements, to be rated on the scale of 1-5, asking for the opinions of students about learning Linear Algebra using graphing calculators. The second part consisted of 5 open – ended questions. The criterion for assessing a "good" attitude in learning Linear Algebra using graphing calculators was the average mark of the first part if not less than 3.5.

Learning, teaching and social norm of the class that learned Linear Algebra using graphing calculators came from the data obtained by observing the teaching, learning Linear algebra using graphing calculators, atmosphere in the classroom and behavior of students in the classroom as observed and recorded by the researcher.

Research results
Hypothesis1
The research results showed that there were 22 students from 28 students, or 78.57% of the total number of students, who got scores of at least 60% of the total score. The statistics Z-test on Hypothesis1 shows Z-value and p-value in Table 1.

Table 1. Z-value and p-value of Z-test on Hypothesis 1.

Total students	Number of passed students	Z-value	p-value
28	22	2.0058	0.0238

From Table1, it can be concluded that at least 60 % of the students who learned Linear Algebra using graphing calculators passed the criterion for achievement at the 0.05 significant level (p-value < 0.05).That is, the result satisfies Hypothesis 1.

Hypothesis2

The data analysis for testing Hypothesis 2 was derived from testing for the normal distribution of the achievement difference of the high achievement group and low achievement group using Lillifors Test. The results are shown in Table 2.

Table 2. Lillifors Test for normal distribution.

Type of group	Number of students	Test value: L	p-value
High Group	9	0.157	0.2
Low Group	14	0.169	0.2

From Table 2. , it can be concluded that the achievement difference of both groups is acceptable as a normal distribution at the 0.05 significant level (p-value > 0.05).

The researcher tested Hypothesis 2 by comparing the difference between the achievement difference of the high achievement group and low achievement group which produced the results as shown in Table 3.

Table 3. Test for equality of variances and t-test for the achievement difference between the high achievement group and low achievement group.

Variable for testing the difference	Levene's F-test for equality of variances	p-value for comparing equality of variances	t-value for the case of equal variances	p-value of the t-test
Achievement difference	0.928	0.346	1.212	0.239

In Table 3, the p-value 0.346 for comparing equality of variances was more than $\alpha = 0.05$, so we can accept that the variances of the achievement difference of the high achievement group and low achievement group were equal. Then we used the t-test for the case of equal variances and found that the p-value 0.239 of t-test was more than $\alpha = 0.05$ leading to the conclusion that the achievement difference between the high achievement group and low achievement group was not significantly different at the 0.05 level.

Hypothesis 3

The results concerning the attitudes of the sample group about learning Linear Algebra using graphing calculators were obtained from a 5-level rating of 15 statements, each statement being a positive statement. The conclusions are follows:

There are 5 statements where the average marks were at the strongly agree level.

- The efficiency of graphing calculators was technologically suitable for learning Linear Algebra.
- The students could save time in computation.
- The students acquired good experience from the opportunity to use graphing calculators.
- The students agreed and were satisfied with using graphing calculators in the examination.

- The students suggested using graphing calculators with the next generation.

There are 10 statements where the average marks were at the agree level.

- The graphing calculator was easy to operate.
- The students enjoyed and did not slack off doing the exercises.
- The students were able to make a conjecture in a short time.
- The students had more time for thinking about problem solving.
- The students were not bored in learning Linear Algebra.
- The students paid attention and gave more time to learning Linear Algebra.
- The students were satisfied to use graphic calculators in learning Linear Algebra.
- If the students could turn back to the past and choose to learn Linear Algebra via graphing calculators or using the traditional methods, they would choose to learn using graphing calculators.
- The students want to use graphing calculators in learning other subjects in mathematics.
- The students suggested promoting the method of teaching mathematics using graphing calculators to other levels and other institutes.

A consideration of all 15 statements showed that the average attitude score was 4.25 which was at the "**agree**" level of rating, and meant that the attitudes of the students in learning Linear Algebra using graphing calculators was good, because the average score was more than 3.5, the criterion for a good attitude. Thus the result satisfies Hypothesis 3 .

Learning, teaching and social norm of the class

Besides the study of achievement and attitudes in learning Linear Algebra using graphing calculators, the researcher paid attention to observing the learning, teaching and social norm of the class in terms of the results of learning Linear Algebra using graphing calculators, the atmosphere of the classroom, together with the students' behavior during learning Linear Algebra with graphing calculators.

The research results showed that the students who learned Linear Algebra using graphing calculators could save a lot of time in problem solving because a graphing calculator was effective in time-consuming computation, where errors easily occur when using only paper and pencil. The students had a lot more confidence in learning mathematics. The instructor was able to teach more and more deeply, since he could save time by using a graphing calculator so that the learners could study various computation methods. The graphing calculator could be used to give leading examples for new topics or new theories so that they were easy to understand and interesting. The mathematics properties were also easily proved by using leading examples so that the learners had the ability to give reasons and learn critically. The learners felt that learning by this method was a good experience for them and suggested that the teaching and learning of Linear Algebra using graphing calculators should be used with other groups and the next generation.

Regarding the social norm (the atmosphere of the classroom in learning Linear Algebra using graphing calculators), the classroom used in this study was an ordinary classroom. In the class, each student would take a graphing calculator from the box in front of the classroom to his/her own seat. This made the classroom look like a computer lab, with graphing calculators serving as computers. During the learning period, students would operate their graphing calculators along with the instructor or work through the activities on their own worksheets, recording experimental results in the same manner, so that it had the air of a dynamic classroom.

The graphing calculators were new technology for the learners, providing a new experience in easy-to-use technology. The students paid greater attention and were excited to have the opportunity to use them. They were eager to learn and were not bored in learning or solving problems in the exercises. They were interested in the graphing calculator screen on their own calculators, and the big screen projector from the instructor's unit in front of the classroom, so they concentrated on learning. They used most of their time for studying problem solving processes without worrying about computation or confidence in the outcome. The atmosphere of the classroom created a good correspondence between the instructor and learners, so that the learners dared to answer questions because they were confident in the answer from the graphing calculators. There were a lot of discussions, arguments and academic opinions that made for happy learning and teaching and without stress. These are the data obtained from the open-ended questions in the questionnaire.

Summary

This research produced the following results:

The result supported Hypothesis 1: more than 60 % of the students who learned Linear Algebra using graphing calculators passed the criterion.

The result did not support Hypothesis 2: the achievement difference between the high achievement group and low achievement group was not significantly different.

The result supports Hypothesis 3: the attitudes of the students in the sample group in learning Linear Algebra using graphing calculators were at the good level.

Discussion

Although the passing criterion for the Linear Algebra course was set high at 60 % of the total score, the result that more than 60 % of the learners passed the course showed that a graphing calculator was effective in learning Linear Algebra and produced high achievement. These good results may be explained by the fact that the learners were interested in learning, were not bored with doing exercises and Home Quizzes in every worksheet and also because of the students' good attitudes in learning Linear Algebra by using graphing calculators, as observed from their behavior and the learners' questionnaires.

The achievements in learning Linear Algebra by using graphing calculators of the high achievement group and low achievement group were not significantly different perhaps because the size of the sample group was too small for division into high and low achievement groups. If the sample size is increased the result may be different. Moreover, the Linear Algebra course used in this experiment is a core course that emphasizes proof, so that part of the achievement in learning involves the subjective question about proof, which is hard for low level basic mathematics students, and a graphing calculator is not helpful in proving. This may have caused the achievements in learning Linear Algebra by using graphing calculators of the high and low achievement groups to be not significantly different.

The data that most usefully support the development of the learning-teaching of mathematics are the students' attitudes in learning using graphing calculators, the results of the learning and teaching, and the social norm of the class. All of these are effective if the learners have good attitudes. Then the learning atmosphere is full of happiness and without stress. The research shows that the students' attitudes in learning Linear Algebra by using graphing calculators is at the good level with a 4.25 average, which indicates that the graphing calculator is useful in learning mathematics.

Suggestions

Suggestions for learning Linear Algebra by using graphic calculators

In this research, the researcher found that the learners had a problem in how to operate the graphing calculators and solve problems when they could not evaluate the result in the case of a wrong input command by the learners. This problem happened only at the beginning of the experiment because the learners had no experience in using graphing calculators. It was a basic usage problem. The problem should be solved by giving 3 hours more time to the learners to practise how to operate the graphing calculators so that the learners will get more used to them. The learners confirmed this problem in the questionnaire by suggesting that the instructor should hand out the user's graphing calculator manual and information on problem shooting so the learners can correct the problem later on after class.

At the beginning and before using a graphing calculator to learn Linear Algebra, the learners should be taught to solve problems by the paper-and-pencil method without graphing calculators and then, when the learners understand, the instructor can let the learners repeat doing the problems by using graphing calculators to shorten the solving time. But graphing calculators cannot be used directly in problem-solving process. The learners must input commands to the graphing calculators themselves without omitting any concept in the problem- solving process. However, the researcher experimented in the first achievement test, which was the subjective test, without using graphing calculators and found that most learners could solve the problems, while some learners computed incorrectly even if they had the correct solving method.

Suggestions for related research on graphing calculators

Since the graphing calculator is a new form of technology in Thailand, there is very little research related to using graphing calculators in learning and teaching. Given that graphing calculators will be cheap for education in mathematics and science, we should be doing more research in various aspects to confirm the effectiveness of the graphing calculator and to increase the use of this technological tool in the learning and teaching of mathematics. For example, there should be research on

1. the results of learning mathematics at the elementary or prathomsuksa level,
2. the results of learning mathematics at the secondary or mathayomsuksa level,
3. the results of learning mathematics in other courses, such as algebra, geometry, calculus, etc.,
4. the scope of learning and teaching mathematics at various levels for maximum benefit.

References

Browning, C. A. (1989). Characterizing levels of understanding of functions and their graphs. **Dissertation Abstracts International, 49,** 2957A.

Dick, T. (1992). Supercalculators: Implications for calculus curriculum, instruction, and assessment. In J. T. Fey (Ed.), **Calculators in Mathematics Education: 1992 Yearbook of the National Council of Teachers of Mathematics** (pp. 145-157). Reston, VA: NCTM.

Hollar, J. C. (1997). The effects of a graphing approach college algebra curriculum on students' understanding of the function concept. **Dissertation Abstracts International, 57**, 2974A.

Kinney, D. P. (1997). The effect of graphing calculator use and the Lesh translation model on student understanding of the relationship between function and derivative in a nonrigorous calculus course. **Dissertation Abstracts International, 57**, 5090A.

McClendon, M. A. (1992). The development of a graphics calculator study guide for calculus students. **Dissertation Abstracts International, 52**, 2450A.

Runde, D. C. (1997). **The Effect of Using the TI-92 on Basic College Algebra Students' Ability to Solve Word Problems.** Research Report (143), Manatee Community College, Florida.

Siskind, T. G. (1995). The effect of calculator use on mathematics achievement for rural high school students. **Rural Educator, 16**(2), 1-4.

Slavit, D. (April, 1994). **The Effect of Graphing Calculators on Students' Conceptions of Function.** Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, April, 1994.

Waits, B.K. & Demana, F. (2000). Calculators in mathematics teaching and learning past, present, and future. In **Learning Mathematics for a New Century** (pp.51-65). Reston: VA: National Council of Teachers of Mathematics.

Wilkins, C.W. (1995). The effect of the graphing calculator on student achievement in factoring quadratic equations. (Mississippi State University, 1995). **Dissertation Abstracts International 56**, 2159A.

Technology Becoming Infrastructural in Mathematics Education

Jim Kaput
Department of Mathematics
University of Massachusetts-Dartmouth

Note to ICME TSG-15 readers: This is a first draft of what amounts to an introduction to the larger issue. It is offered as an early framework for understanding the issues raised by our plenary speakers. Time constraints prevent the inclusion of references to the several literatures that it draws upon. I look forward to having its ideas challenged and improved by our interactions.

1. The Goals and Challenges of Accounting for Technology's Impact

1.1. The Goals of the Paper and Related Complications

The goal of this paper is to provide a framework that helps us understand the gradual, manifold evolution of technology's[1] roles in mathematics education. The underlying idea is that the changes over the longer term amount to a process of technology becoming "infrastructural." Our first task is to describe what we mean by "infrastructural," and second, using this characterization, to analyze where we are in the long process of technology's symbiotic relation with mathematics and mathematics education. Our outline of the subtleties, challenges and complexities below is intended to serve the ultimate objective, not realizable in any particular paper, let alone an introductory paper such as this, to render intelligible the long term interactions between technology and mathematics and mathematics education.

But the situation is both complex and subtle if one takes the point of view, as I do, that *technologies and tools co-constitute both the material upon which they operate and the conditions, particularly social conditions, within which such operation occurs*. This means that we do *not* take an essentialist view that mathematics somehow exists independently of tools and technologies, platonically, socially shared, or otherwise, and that technologies and tools infiltrate and transform the mathematics over time.

Nonetheless, the rate of change of technological tools has become very high in the digital age, and the inherited corpus of shared mathematical knowledge that was produced in interaction with pre-digital technologies is large and stable, particularly that part of it that intersects with schools and schooling – the mathematics of mathematics education. The same can be said of the *practices* of mathematics education, which are embedded in and part of a very stable system of social practices and institutions. Hence, despite the fact that we do not take an essentialist view, we find ourselves in the position of analyzing the impacts of new technologies on older mathematics and educational practices and institutions. But in addition, our account must

---

[1] For brevity, unless otherwise specified, we use the term "technology" to refer to electronic, mainly digital, information technologies, both hardware and software, in their many forms, including computers of various sizes and configurations, visual display and communication systems, memory storage systems, and so on – the commonplace use of "technology" in mathematics-education. Of course, it is essential, and indeed this is one focus of this paper, to recognize the greater generality of the term across space and time.

include provision for the rapidly evolving new mathematics that is generated by new digital tools and technologies, and it must deal with the fact that this mathematics influences and helps make possible yet new tools and technologies, compounding the interaction.

Hence, for purposes of analysis, we will distinguish "old" from "new" mathematics, that which existed before and after the advent of digital technologies, respectively. Of course, the temporal boundary is not sharply defined, but crosses a span of roughly 50 years covering the second half of the 20th century, with perhaps the sharpest transition being between the 1960's and 1980's. The epistemic boundary is likewise irregular. After all, the iterative and highly visual mathematics of dynamical systems is surely "new" and rapidly growing in the computational medium, especially since the latter 1970's, but has its roots at the turn of the previous century, in the work of Poincare especially.

Finally, while one can analyze the interaction from a collective, historical perspective, in mathematics education we must attend to the matter of how mathematics comes to be known through acts of teaching and learning, within schools, classrooms and individuals, in the short term. Therefore, a full account must deal with multiple time-scales and social scales.

1.2. Challenges in Accounting for Technology's Interactions with Mathematics Education

1.2.1. The Epistemic Complexity of Mathematics

One challenge is the complexity of mathematics itself. First, there are the many kinds of mathematics, in terms of mathematical domains, whose relationships with digital technologies vary – analysis, algebra, geometry, number theory, the mathematics of data, applied vs. "pure" mathematics, and so on. And second, there are important epistemic dimensions within mathematics. One dimension concerns the kinds of objects and relations that are regarded as mathematical within the various mathematical domains, a second concerns the kinds of representations and languages by which the former are represented and acted upon, many of which are shared across domains, and a third concerns the forms and modes of justification and truth. Each of these dimensions interacts with technology in different ways.

1.2.2. Two Different Kinds of (Software) Technology Affordances: Representation and Communication

In mathematics education, we have long examined the *representational* affordances of technology: supporting a partnership with users in syntactical manipulation of representations, offloading computational duties (e.g., root-finding, matrix diagonalization) to technology, adding new actions to traditional notation systems (e.g., actions on coordinate graphs), supporting new interactive notation systems including programming languages and spreadsheets, linking notation systems, new or old, and so on. Within this affordance, some representational uses are more directly computational such as symbolic manipulation or iterative computation than expressively representational or visual. But we shall group these both together within the large representational affordance. We also include human-computer-interaction interfaces within this category, most especially the use of "direct manipulation" systems as occur in new software environments.

On the other hand, computational power has increasingly been used in support of human *communication* in the world at large, both in a hidden infrastructural way, as when computers control the flow of information inside networks or telephones, or even within less interactive technologies such as television, and in the visible interactions that occur when we send messages or computationally defined objects to one another. The internet and its grand application, the World Wide Web, are a conspicuous set of examples that mathematics education has exploited in a variety of ways, ways that are not especially domain specific to mathematics. However, we will discuss additional within-classroom communication affordances that are distinctly specific to mathematics education.

A hybrid computational affordance involves the importing of physical data into the computational medium ("MBL" – Microcomputer Based Laboratory), as when motion sensors are linked to computational devices and position, velocity or acceleration data are represented on the device. Indeed, this process can be reversed, as with "LBM" – Line Becomes Motion as exploited by Nemirovsky and colleagues. For simplicity's sake, we will treat these as communication affordances.

We should acknowledge a third major use of technology, the storage and manipulation of data, databases. For simplicity's sake, since this use is usually embedded within the other two uses in mathematics education, we will treat it as an implicit use and not define a third explicit affordance – at least for this draft.

Finally, we should note that the above description of technology is very different from one based on *use-patterns*, classically captured in the "tool, tutor, tutee" analysis. The first is transparent, although is a bit ambiguated by connectivity, the second refers to technology's use in systems where interactivity takes the form of pedagogically organized assistance, and the third refers to its use as a programming and design environment. Over time, these uses have tended to be combined in various ways and seldom occur in pure forms, a trend likely to accelerate with the increase availability of connectivity, which makes resources, including tools and pedagogical resources, available in new ways.

1.2.3. Three Technology Hardware Dimensions: Device Types, Display Technologies & Network Technologies

We distinguish three dimensions of technology available to mathematics education, physical forms of computational device types, display technologies, and network technologies. Computational devices with which people have direct interaction include calculators of various capacities ranging from 4-function numerical calculators with or without memory capacities to scientific calculators, graphing calculators, tablet and PDA like devices, laptop computers, desktop computers, and sophisticated workstations. Display technologies range from the built-in screens of the computational devices to monitors and whole-class display systems. Networks include both wired and wireless, and each varies in scales from a few devices within a room or site to the internet, and their topologies vary to include server-client relations to point-to-point communication, and virtually any mix of these.

1.2.4. Trends Within & Across Technology Hardware Dimensions

Within each of these dimensions, we have seen relatively clear trends. Except for the simplest calculators, with the processing resources continuing to grow while simultaneously becoming more compact according to Moore's Law, software affordances have trended downward from larger to smaller devices as smaller devices become capable of hosting independently produced software (e.g., graphing calculators with flash RAM capability thereby becoming flexible computational devices), and as a result software is increasingly appearing across multiple device types.

Display devices with high resolution are becoming cheaper and more ubiquitous, particularly as visibly shared displays in classrooms. Display devices are increasingly used as input devices via touch-screen on PDA-type and tablet systems, and sometimes combined with large displays as in interactive whiteboards.

Networks have increasingly become more ubiquitous, so that computational devices are seldom used in isolation, and wireless connectivity is moving into schools and classrooms, replacing wired networks (see the last section below). Moreover, not only network systems, but technology in general, is becoming ever "easier" to use, with increasing deployment of self-configuring systems, user-configurable devices and convergence of interface styles. However, "easier" is a relative term and has much to do with our notions of "infrastructure" as to be discussed further below.

Combinations of these trends across device-display-network dimensions have made possible ever increasing availability and manipulability of digital objects of all kinds across devices and networks, most notably video and audio objects. To date, these "heavier" objects, in terms of bit-weight, have not been centrally utilized in mathematics education.

1.2.5. Managing the Complexity of Mathematics Education – Some Arbitrary Category Choices (unfinished section)

We must inevitably deal with the complexity of the field of mathematics education, and our strategy for managing this complexity is to describe the field in terms of "dimensions." In a sense, defining these cannot avoid being arbitrary and ad hoc, including decisions on how fine a decomposition to use. Further, they are in no way independent or "orthogonal." Indeed, the interconnectedness of these dimensions is a basic feature of the field as a human endeavor.

We can begin with the three major activities of teaching, learning, and assessment. And the organization of mathematics content to be taught and learned – curriculum. But then there is also teacher education, and also the political-social administration of mathematics education in schools. There is also the economic side of curriculum material and its distribution. But, as introduced above, there is the all-important epistemological dimension, the dimension within which we situate mathematics content itself. However, this dimension can be further decomposed into the various branches of mathematics. For purposes of simplicity, we keep the categories simple:

2. Meanings of "Infrastructure" and Related Issues of Literacy Communities and Communities of Practice

For the record, I recall the dictionary definitions:

American Heritage: 1. An underlying base or foundation especially for an organization or system. 2. The basic facilities, services, and installations needed for the functioning of a community or society, such as transportation and communications systems, water and power lines, and public institutions including schools, post offices, and prisons.

The word "infrastructure" has been stretched over the years to include not only material supports for activity, but also social systems at different size scales, sometimes referred to as communities of practice in the sense of Lave and Wenger. As we shall discuss further, technology plays multiple roles as an infrastructure supporting other infrastructures, both physical and otherwise.

Our intuitive starting point for understanding the process of technology becoming infrastructural is dual: First from the phenomenological perspective of individuals, the *experience* of technology, transitioning from strange to familiar to invisible. Second, from an external, analytic perspective. From this perspective we employ several analytic frameworks in an eclectic way, particularly as we attempt to keep in mind the multiple time-scales in which technological change can be described, ranging from months to millennia.

2.1. The Technology-Driven Evolution of Technology-User Communities

We need to keep in mind the sociological side of "infrastructurality" partly captured in diSessa's notion of technology and literacy infrastructures (in his book, *Changing Minds*), which he links to literacy communities defined by their literacy practices – he uses romance novel readers as an example, who typically read this particular genre while commuting to and from work, and who are predominantly female. However, as a community, they are passive users compared to the active users of most mathematics or mathematics education software, where active participation in a practice is an intrinsic property of membership. This is the case whether one uses the technology as an interactive tool or as a medium in which one designs and builds interactive artifacts (technology as "tutee"). Hence we merge the two notions – literacy community and community of practice – into *technology user community*.

CAS users comprise an example of a technology user community among technology users in Mathematics Education. For them, CAS software is an important infrastructure, and for analysts, they are a fairly well-defined community whose features, especially its induction processes and relations with other communities, are significant factors in an account of technology becoming infrastructural.

Another such community is comprised of Dynamic Geometry users. In the past, while the software systems that they took as infrastructural were independent from the CAS systems, the overlap with the CAS community has been defined by professional and educational interests – a particular person might use both systems and identify with each community, but use each system for different purposes. However, now that the respective underlying software functionalities of CAS and Dynamic Geometry are increasingly overlapping, their user communities may merge. But more importantly for our purposes, *the emergence of a common infrastructure uniting two communities is deep evidence of technology becoming infrastructural at the epistemic level*. As a result of the software systems growing together into a new, more integrated whole, the practices of the Dynamic Geometry community, whose mathematics domain embodies certain kinds of objects and relations, as well as certain styles of justification and proof, inevitably interact with those of the CAS community, whose objects and relations become transformed when viewed as geometric ones. Further, the styles of exploration, justification and proof in one domain interact with those of the other. In a sense the seed planted by DesCartes blooms and bears fruit in the computational medium, and the respective technology user communities unify in a more unified mathematical practice.

2.2. Necessity vs. Convenience as a Factor in Infrastructurality

Another factor in technology's being infrastructural is the matter of necessity, the fact that when the technology is fully infrastructural for a particular use, its use is necessary and not optional. Thus, for example, the study of the behavior of certain dynamical systems and their visual features requiring millions of iterations of functions requires computational technology in ways that plotting the intersection of two low-degree (say quadratic) polynomials for an approximate solution does not. In effect, the objects of interest literally do not exist without the technology in the first case, while the technology is a convenience in the second. Of course, "convenience" shades into necessity and at a certain point technology creates new forms of practice and hence new mathematics.

2.3. The Issue of Interactivity and Agency

Over time, thanks to more powerful processing and increasingly "natural" input modes utilizing natural (or at least, widely occurring) human capacities, software has become ever more interactive in the sense of responding to human-user input in ever wider communication bandwidth and ever shortening feedback cycles, approaching real-time feedback in many circumstances, e.g., in most direct manipulation interfaces. Furthermore, what we have broadly described as "tools" are increasingly responsive to context and individual use patterns. While we commonly describe this change as "increasing intelligence" of the tools, it may be at least as fruitful to think of the tools as *increasing in agency*. We have tended to ascribe agency only to humans (in our theorizing, but not in our interactions with technology, where we regularly treat the computational device as having a mind and even a personality of its own). But once we ascribe agency to tools and technologies, then we have an opportunity to rethink our relation to technology in terms of *partnership*. While the notion of intelligence distributed across tool and user is far from new, going back at least to Dewey, by adding the notion of tool-agency, we may be able to examine our relation with technology in a deeper way that helps us understand technology's impact in mathematics education in more systematic ways.

Recent work by David Shaffer has taken this stronger form of distributed intelligence to include interacting agency – in effect, the new unit of analysis is the partnership, which he refers to as "toolforthought" – deliberately combining words to emphasize the unicity of the entity under consideration. A toolforthought may be regarded as a cross between a cyborg, a computer-augmented human and a human-enhanced tool. This orientation may reveal patterns in the evolving relations between increasingly powerful tools and the body of mathematical knowledge and skill that are taken as the educational agenda of mathematics education. It may also help us understand more fully the meanings of "powerful" in the preceding sentence. The next section is intended to take a step in this direction.

2.4. Regularities in the Changes in Distribution of "Skill" Across Tool and User

On one hand, we are familiar with the processes of "handing off" computations and syntactically-defined operations to software, or having the system provide or link to a new representation, and hence the redistribution of operational skill from human to machine. On the other, we are as yet unclear on the new skills that this handing-off requires, especially when the software, as with the typical CAS, can provide multiple representations of the operations, their product, or both. This was a major concern at earlier ICME Technology meetings [see, for example, Fey, J. (1989). Technology and mathematics education: A survey of recent developments and important problems. *Educational Studies in Mathematics, 20*, 237-272)].

My first point is that, from a longer term perspective as well as from any of several theoretical frameworks, this tool-user relationship issue is a particular species of a broad research issue that occurs across all areas of education, not merely mathematics. It is sometimes framed as a language or culture vs. individual learning and/or development issue, sometimes as a cultural anthropological issue, sometimes as a history of culture issue, sometimes as an epistemological issue, and so on. I have long taken the view that in mathematics some of the most important and generative advances are representational – the means by which people think, compute, communicate, etc. In particular, across history regularities in procedures come to be encoded (either deliberately or through natural selection-like processes) in notation systems and actions upon these.

As Shaffer and I argued in a 1999 *ESM* paper extending the work of the evolutionary psychologist Merlyn Donald, the human mind was externally augmented by static records based on writing systems (and as Moreno points out, human externalization and crystallization of action occurred much earlier, e.g., with tally systems), and then, several thousand years later in mathematics by syntactically structured *operative* notation systems that, in partnership, support human action, e.g., the algebra representational infrastructure, as well as the standard arithmetic representational infrastructure. Then, in the 20th century, operative systems became autonomously executable within the computational medium. Indeed, the more general idea due to Turing and von Neumann is that of "program," which was applied to encode and extend previously constructed operative notation systems, leading to, among many other things, CAS systems. Such systems call upon a different kind of partnership between human and tool, or, if you prefer the strong version, they generate a new kind of toolforthought object.

While the broad pattern of externalization and partnering with external tools is evident, the details are very complex and involve changes in cognitive and perceptual activity as learning occurs within individuals and across history. One needs to learn to work as a partner with ever more agency-laden tools, tools that themselves carry out increasingly complex processes, processes that themselves were once carried out by humans, sometimes requiring considerable mental effort and skill. The same thing happened in the 15th century with the adoption of the base-ten placeholder system for numbers and the algorithms that were built upon it and that continue to dominate elementary school mathematics. In a deep sense, the mathematical experience is continually changing. So, in particular, our task as educators and designers of educational experiences is to design for an ever-changing partnership rather than to worry over the loss of human skill as tools increase in capacity.

2.5. The Rate of Change Problem: Changes in Representational Infrastructures vs. Changes in Surrounding Institutional Infrastructures

Thanks to the plasticity of the computational medium and its ability to fuel its own change as its own infrastructure, the rate of change is accelerating – the change is more like exp(exp(x)) than exp(x) or polynomial change. Our challenge, then, as mentioned earlier relative to the essentialist view of mathematics, relates to the problem of time-scales of change. If representations and partnerships change at a rate of change faster than the rate of change of the surrounding institutions, then we have a major problem. Because educational institutions, (schools, assessment systems, teacher education systems, and so on) are inherently conservative social systems, this is exactly what we have!

But the problem is more complex because some of the most important changes in technology, especially computational technology, do not occur in the straight-ahead direction, but appear from unexpected directions. Who saw the WWW coming? This leads us to what I believe is another important change that is about to descend upon us related to the communication affordance of technology, which is likely to interact in unpredictable ways with the representational affordance.

2.6. The Communication Technology Affordance and Wireless Classroom Connectivity

A major change is underway that, because it takes place at the communicative heart of the epicenter of mathematics education – the classroom – and involves the teacher in a central way, is likely to yield profound consequences. Thanks to technologies developed outside education (as is usually the case), the ability to wirelessly link devices of various kinds and, as noted earlier, the ability to scale software and data across device types, an entirely new universe of classroom possibilities is emerging. This is the focus of my own recent work – with my colleagues Stephen Hegedus and Jeremy Roschelle, as well as more distant colleagues, Walter Stroup, Uri Wilensky, and others. In effect, the communication affordance is moving from outside the classroom to inside the classroom.

2.6.1. Key Issues in Classroom Connectivity and Its Infrastructural Role

The major new ingredients that Hegedus, Roschelle and I have been studying are (a) the mobility of multiple representations as reflected in the ability to pass these bi-directionally and flexibly between teacher and students and among students, using multiple device-types, (b) the ability to flexibly harvest, aggregate, manipulate and display representationally rich student constructions to the whole classroom, and (c) thanks to the at-handedness of hand-helds, to do all of this in ways that respect and build upon naturally occurring social and participation structures. We are thus able to engineer and implement a surprising array of new activity structures in concert with the mathematics to be taught and learned.

We see classroom connectivity (CC) as a critical means to unleash the long-unrealized potential of computational media in education, because we see its potential impacts as direct and at the communicative heart of everyday classroom instruction. We are now beginning to build insight into how those new ingredients, in combination, may provide the concrete means by which that potential may be realized, because they may, in fact, help constitute the first truly educational technologies, intimately situated within the fundamental acts of active teaching and active learning. This embeddedness may indeed be more profound than we initially recognized, because these ingredients resonate deeply with broader views of learning as participation and no longer fit within a "learning in relation to a machine" (large or small, in the lab, classroom or even your hand) view of educational technology. Indeed, the paradigm is shifting towards one where the technology serves not primarily as a cognitive interaction medium for individuals, but rather as a much more pervasive medium in which teaching and learning are instantiated in the social space of the classroom. We deliberately choose "instantiated" ahead of "situated" because we have repeatedly seen mathematical experience emerge from the distributed interactions enabled by the mobility and shareability of representations (see below).

The student experience of "being mathematical" becomes a joint experience, shared in the social space of the classroom in new ways as student constructions are aggregated in common representations – in ways reminiscent of, but distinct from those of a participatory simulation as studied by Resnick, Stroup, and Wilensky. Cognitive activity is distributed in the socio-material space in the sense of Hutchins. Similarly changed is how students interact mathematically with each other and their teacher, and, critically, how their personal identity is manifest in their shared mathematical experience in the classroom.

For *teachers*, it changes the nature of teaching by fundamentally altering how participation structures can be defined and controlled, how attention can be managed, how information flows and can be displayed, and how pedagogical choices and moves are made in real time. Adding to the richness of the picture is the fact that the continuing evolution in the underlying technological-communication affordances affects all these factors in important ways.

For *researchers*, the ground may shifted in a profound way, challenging our categories of phenomena. The fulcrum of the balance between "individual-cognitive" and "social-distributed" may have shifted. Semiosis may have become a new kind of operation, applying to shared objects in newly sharable contexts. [incomplete – need finish section]

2.6.2. The Need for Research in Classroom Connectivity

But the potential of CC can be realized only if we understand it sufficiently to inform the design and improvement of (a) its technologies, (b) classroom activities, teaching practices and forms of assessment that optimally exploit it, and (c) the preparation and support of teachers to utilize this new constellation of technologies, activities, practices, and assessments. This will require a new, highly interdisciplinary domain of educational research, one that is now in its early stages, uncovering the new phenomena to be investigated, formulating issues, descriptive languages, candidate theories, and research agendas, and building research communities to extend and elaborate the inquiry.

2.6.3. Starting Point Questions in Classroom Connectivity

A large number of new phenomena need to be studied. (I hope to provide some concrete examples in my presentation.) We have defined three Opportunity Spaces generated by CC and pose central questions associated with each.

Learning and Activity Structures: What new activity structures are possible and appropriate that exploit CC across diverse device-types, that increase learning of traditional topics, that render new topics more accessible, and that increase breadth of student participation and intensity of engagement?

Teaching and Pedagogy: How does CC impact teacher decision-making and pedagogical options, both positively and negatively, and how can teachers learn to use CC to maximal advantage?

Assessment, Classroom Management and Information Flow: How can teachers use CC and analytic tools to exploit what we know about student thinking and learning in order to actively diagnose and efficiently respond to student thinking on a regular basis? What kinds of tools can facilitate the flow, organization and display of information in the classroom?

Two sets of questions cross-cut these:

Questions of Representation: What kinds of representations and uses of such (e.g., on single devices, distributed across devices, aggregated or not, etc.) support optimal student learning and participation, teacher decision-making and activity-design, interpretation of assessment data, and information flow and display?

Questions of Technology: Which combinations of technological characteristics within and across devices (e.g. screen and physical interface, communication capacity, portability, processing power, etc.), networks (e.g., peer-peer, server-based, pull vs. push communication, etc.) or software (e.g., common data-structures and interfaces across devices, control at-a-distance, etc.) enable ease, fluency, and effectiveness of mathematical learning, teaching, assessment, and information flow?

We are actively pursuing these issues at this time. It has become abundantly clear that the evolution of this new technology infrastructure is driving a need for new theoretical frameworks and constructs as well as new research organization. In a sense, this is technology infrastructure reflexively redefining the conditions of its own study.

3. Unfinished Business

Most of it. To apply the matrix resulting from the categories defined in the first section. In particular, we can organize our analyses of the degrees to which technology has become infrastructural using the cells of a matrix whose vertical columns are defined by the two technology affordances and whose rows are defined, in groups, by the various groups of categories developed in the first section. Of course, it would be preferable to use a higher dimension matrix, but, for now we will limit ourselves to two dimensions. I hope that we can use the matrix to view the ICME papers submitted to date in terms of what they have to say about technology becoming infrastructural in mathematics and mathematics education.

Mathematical thinking and technology: some views on their co-evolution

Luis Moreno Armella
Matemática Educativa, Cinvestav, Mexico.

A pre-historical perspective on technology

Human evolution is coextensive with tool development. In a certain sense, human evolution has been an *artificial* process as tools were always designed with the explicit purpose of transforming the environment. And so, since about 1.5 millions years ago, our ancestor Homo Erectus designed the first stone tools and took profit from his/her voluntary memory and gesture capacities (Donald, M. 2001) to evolve a pervasive technology used to consolidate their early social structures. The increasing complexity of tools demanded optimal coherence in the use of memory and in the transmission, by means of articulate gestures, of the building techniques. We witness here what is perhaps the first example of deliberate teaching. Voluntary memory enabled our ancestors to engender a mental template of their tools. Templates lived in their minds, granting an objective existence as abstract objects even before they were *extracted* from the stone. That objectivity resulted from the identification of the tool with its complex production (a sequential process kept alive thanks to voluntary memory) and its meaning turned visible by the shared uses by the community. Thus, tool production was not only important for plain survival, but also for broadening the mental world of our ancestors –introducing a higher level of objectivity.

The actions of our ancestors were producing a *symbolic* version of the world: A world of intentions and anticipations they could imagine and *crystallize* in their tools. What their tools meant was the same as what they *intended* to do with them. They could *refer* to their tools to *indicate* their *shared* intentions and, after becoming familiar with those tools, they were looked as *crystallized* images of all the activity that was embedded in them.

We suggest that the synchronic analysis of our relationship with technology, no matter how deep, hides profound meanings of this relationship that coheres with the co-evolution of man and his tools. It is then, unavoidable, to revisit our technological past if we want to have an understanding of the present.

The Royal road to Mathematics: Ancient Counting Technologies

Evidence of the construction of one–to–one correspondences between arbitrary collections of concrete objects and a *model set* (a template) can already be found between 40000 and 10000 B.C. For instance, hunter-gatherers used bones with marks (tallies). In 1937, a wolf bone dated to about 30000 B.C. was found in Moravia (Flegg, G.: *Numbers: Their history and meaning*, Penguin, 1983). This reckoning technique (using a one-to-one correspondence) reflects a deeply rooted trait of human cognition. Having a set of stone bits or the marks on a bone as a *modeling set* constitutes, up to our knowledge, the oldest counting technique humans have designed. The modeling set plays, in all cases, an instrumental role for the whole process. In fact, something is crystallized by marking a bone: The *intentional* activity of finding the size of a set of hunted pieces, for instance, or as some authors have argued, the intentional activity of computing time.

The modeling set of marks, plays a role similar to the role played by a stone tool as both mediate an activity, finding the size, and both crystallize that activity. Between 10000 and 8000, B.C. in Mesopotamia, people used sets of pebbles (clay bits) as modeling sets. This technique was inherently limited. If, for instance, we had a collection of twenty pebbles as modeling set then, it would be possible to estimate the size of collections of twenty or less elements. Nevertheless, to deal with larger collections (for instance, of a hundred or more elements), we would need increasingly larger models with evident problems of manipulation and maintenance. And so, the embodiment of the one-to-one technique in the set of pebbles inhibits the extension of it to further realms of experience. It is very plausible that being conscious of these difficulties, humans looked for alternative strategies that led them to the brink of a new technique: the idea that emerged was to replace the elements of the model set with clay pieces of diverse shapes and sizes, *whose numerical value were conventional*. Each piece *compacted* the information of a whole former set of simple pebbles —according to its shape and size. At that age (about 8,000 B.C.) hunting and gathering were replaced with agriculture and due to this new activity problems of storage and distribution of food became much more complex (D. Schmandt-Besserat, 1978). The pieces of clay can be seen as embodiments of pre-mathematical symbols. Yet, they lacked rules of transformation that allowed them to constitute a genuine mathematical system.

Much later, the consolidation of the urbanization process (about 4000 B.C.) demanded, accordingly, more complex symbol systems. In fact, the history of complex arithmetic signifiers is almost determined by the occurrence of bullae. These clay envelopes appeared around 3500-3200 B.C. (Schmandt-Besserat 1978). The need to record commercial and astronomic data led to the creation of symbol systems among which mathematical systems seem to be one of the first. The counters that represented different amounts and sorts —according to shape, size, and number— of commodities were put into a bulla which was later sealed. And so, to secure the information contained in a bulla, the shapes of the counters were printed on the bulla outer surface. Along with the merchandise, producers would send a bulla with the counters inside, describing the goods sent. When receiving the shipment, the merchant could verify the integrity of it.

A counter in a bulla *represents* a *contextual* number — for example, the number of sheep in a herd; not an abstract number: there is five of something, but never *just five*. The shape of the counter is impressed in the outer surface of the bulla. The mark on the surface of the bulla *indicates* the counter inside. That is, the mark on the surface keeps an *indexical relation* with the counter inside as its referent. And the counter inside has a *conventional* meaning with respect to amounts and commodities. It must have been evident, after a while, that *counters inside were no longer needed;* impressing them in the outside of the bulla was enough. That decision altered the semiotic status of those external inscriptions. Afterward, instead of impressing the counters against the clay, scribes began using sharp styluses that served *to draw* on the clay the shapes of former counters. From this moment on, the symbolic expression of numerical quantities acquired an infra-structural support that, at its time, led to a new epistemological stage of society. Yet the semiotic contextual constraints, made evident by the simultaneous presence of diverse numerical systems, was an epistemological barrier for the mathematical evolution of the *numerical ideographs*. Eventually, the collection of numerical (and contextual) systems was replaced by **one** system (Goldstein, 1999). That system was the sexagesimal system that also incorporated a new symbolic technique: numerical value according to position. In other words, it was a *positional* system. There is still an obstacle to have a complete numerical system: the presence of zero that is of primordial importance in a positional system to eliminate representational ambiguities. For instance, without zero, how can we distinguish between 12 and 102? We would still need to look for the help of context.

The work of Cantor on cardinality of infinite sets showed that the last step in the use of one-to-one correspondence with a modeling set had not been taken. By his time, the level of abstraction made possible by mathematical symbolic technology was higher than ever before. This was a level of abstraction and organization of knowledge that cohered with paper and pencil technology.

One virtue of the previous narrative apart from its present simplicity is that it enables us to see the co-evolution of symbolic technology and mathematical thinking. Mathematical objects result from a sequence of crystallization processes that, at a certain level of evolution, has an ostensible social and cultural dimension. As the levels of reference are hierarchical the crystallization process is a kind of recursive process that allows us to state:

Mathematical symbols co-evolve with their mathematical referents and through their semiotic embodiments make possible the shared existence of these mathematical entities among the communities that take them as shared.

Lodging a new technology: Mathematics and computing tools

In what follows, we try to articulate some reflections regarding the presence of the computational technologies in mathematical thinking; mainly from the perspective of education. It is interesting to notice that even if the new technologies are not yet fully integrated within the mathematical thinking, their presence will eventually erode the mathematical way of thinking. The blending of mathematical symbol and computers has given way to an *internal mathematical universe* that works as the reference fields to the mathematical signifiers living in the screens of computers. This takes abstraction a large step further.

According to Balacheff & Kaput (1996), the main impact of information technology on educational systems is epistemological and cognitive, because it has contributed to the production of a new form of realism in mathematical objects. This new form of realism depends on the interpretative resources provided by the socio-cultural environment.

Thus, technology has the power to become an educational agent for change but this process of change is complex.

The virtual versions of mathematical objects produce the sensation of material existence, given the possibility of changing them where they manifest themselves, that is, on the screen. Students' growing familiarization with computational tools allows these tools to be transformed into mathematical *instruments* (Guin & Trouche, 1999; Rabardel, 1995) in the sense that computational resources are gradually incorporated into the student's activity. For example, when secondary school students are asked to explore the relationships between the inscribed angle in an arc and the corresponding central angle, we see two behaviors in the classroom: students remain immobilized by the question (we think this is because they are not able to mobilize their expressive resources) or, when they have computational resources at their disposal (for example, dynamic geometry), they are led to draw up comparative tables between angles and to eventually realize that that the central angle is "nearly double" the inscribed angle in the same arc. The students' strategy, taking the inscribed angle from the central angle is possible thanks to the expressive power the students acquire through the computational tools. In the absence of these, as we have already mentioned, it is not feasible for students to carry out the numerical comparison between the angles and to establish a conjecture, nor are they capable of producing a formulation associated with their explorations and express it in the language of the computational medium in which they are working. The computing environment is an *abstraction domain* (Noss & Hoyles, 1996), which can be understood as a scenario in which students can make it possible for their informal ideas to begin coordinating with their more formalized ideas on a subject. In the example of dynamic geometry, we can put it this way: The exploration of drawings and of their properties gives rise, through the semiotic mediation of dynamic geometry, to the recognition of a system of geometric relationships, which in the final analysis constitute the geometric object.

One of the aims of this research is to understand how the implementation of the new technology should be conducted. We know that the first stage could entail working within the framework of a pre-established mathematical thinking. At that point, it becomes fundamental to understand the nature of knowledge that emerges from their interactions with those mediating tools. In other words, it is important to understand the epistemological role of the new tools when they are lodged into a previous mathematical thinking infrastructure.

Working with computational tools leads us to face their incorporation from two different angles (Berger, 1998): as *amplifying* tools and as *cognitive re-conceptualizing* tools. These amplification and re-conceptualization processes can be illustrated in the following way: The amplification process is similar to the function of a magnifying glass. Through this lens, we can enlarge objects visible at first sight. Magnification does not change the structure of the objects that are being observed, however, on the other hand, the reorganization process can be compared to the act of seeing through a microscope. The microscope allows us to observe what is not visible at first sight and, therefore, to enter a new plane of reality. In this way, the possibility of studying something new and of accessing new knowledge arises.

Computing environments provide a window for studying the evolving conceptions –caused by the presence of these new tools. This can be done analyzing the use of these tools to generate knowledge within a computational environment.

Our work with computational tools has led us to consider the phenomenology one can observe on the screens of calculators and computers. The screen is a space controlled from the keyboard, but that control is one of action at a distance. The desire to interact with virtual objects living on the screen provides a motivation for struggling with the complexities of a computational environment (Pimm, 1995).

During the time that passes while the graph is being drawn on the screen, the student observes the characteristics of the function that are reflected in its construction. We propose, therefore, that the student has the opportunity to transform the graph into an object of knowledge. This is similar to what the Greeks did with writing. They used the writing system not only as an external memory but also as a device to produce texts on which to reflect.

At first, students might make some observations *situated* within the computational environment they are exploring, and they could be able to express their observations by means of the tools and activities devised in that environment. That is the case, for instance, when the students try to invalidate (e.g., by dragging) a property of a geometric figure and they are unable to do so. That property becomes a theorem expressed via the tools and facilitated by the environment. It is an example of *situated proof.*

A situated proof is the result of a systematic exploration within an (computational) environment. It could be used to build a bridge between situated knowledge and some kind of formalization. Students purposely exploited the tools provided by the computing environment to explore mathematical relationships and to "prove" theorems (in the sense of situated proofs). As a new epistemology emerges from the lodging of these computational tools into the heart of today's mathematics, we will be able to take off the quotations marks from "prove" in the foregoing paragraph.

Ruler and compass provided a mathematical technology that found its epistemological limits in the three classical Greek problems (trisection of an angle, duplication of the cube, and the nature of π). Ruler and compass embody certain normative criteria for validating mathematical knowledge. And more general, they are an example of how an expressive medium determines the ways to validate the propositions that can be stated there.

Now we can ask: What kind of propositions and objects are embodied within dynamical mathematical environments? The way of looking at the problem of formal reasoning within a dynamical environment is of instrumental importance. What we propose as a *situated proof* is a way to deal with a transitional stage. We cannot close the eyes to the epistemological impact coming from the computational technologies, unless we are not willing to arrive at new knowledge but only at *new education*. An impossible goal, indeed.

References

Balacheff, N., & Kaput, J. (1996). Computer-based learning environment in mathematics. In A. J. Bishop et al. (Eds.), *International handbook of mathematical education* (pp. 469-501). Kluwer Academic Publishers, Dordrecht.

Berger, M. (1998). Graphic calculators: An interpretative framework. *For the Learning of Mathematics*, 18(2), 13-20.

Donald, M. (2001). *A Mind so rare*. Norton, New York.

Flegg, G. (1983). *Numbers: Their history and meaning*, Penguin.

Goldstein, C. (1999). La naissance du Nombre en Mesopotamie. *La Recherche, L'Univers des Nombres (hors de serie).*

Guin, D. & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: the case of calculators. *International Journal of Computers for Mathematical Learning,* 3, 195-227.

Noss,R. & Hoyles, C. **(1996).** *Windows on mathematical Meanings. Learning Cultures and Computers*. Dordrecht: Kluwer Academic Publishers.

Pimm, D. (1995). *Symbols and meanings in school mathematics*. London: Routledge.

Rabardel, P. (1995). *Les hommes et les technologies. Approche cognitive des instruments contemporains*. Paris: Armand Colin.

Schmandt-Besserat, D. (1978). *The Earliest Precursor of Writing*. Scientific American (pp. 39-47).

The technological presence: shaping & shaped by learners

Richard Noss & Celia Hoyles
Institute of Education, University of London

We start with the assertion that there has been a persistent failure within the mathematics education community to theorise adequately the complexity of supporting learners to develop a fluent and effective relationship with technology in the classroom — by 'effective' we mean helping learners engage with, develop and articulate understandings of mathematical procedures, structures and relationships *through* the technology and according time and status to this process[1]. This failure has, at least in part, stemmed from the fact that it has not been sufficiently acknowledged that mathematical knowledge can be *transformed* by the computer's presence – and even where such acknowledgement exists, the implications of this transformation have not often been thought through at either the theoretical or the practical levels.

Of course this transformation of meanings is not always significant. In a recent review of technology in mathematical education (Hoyles & Noss, 2003), we distinguished two categories of software, programmable microworlds and expressive tools, where such transformation *is* significant and we highlighted the ways in which digital technology shapes and is shaped by its incorporation into mathematical learning and teaching environments. From our review of work with these software, it emerged that tools do indeed shape learning, but they do so in often unpredicted ways. Furthermore, apart from its unsurprising dependence on tasks and activity structures, research with programmable microworlds suggests that learning is highly sensitive to small changes in technologies, and that learning also shapes the tools. Thus the design of tools and learning has tended to co-evolve.

The transformation of mathematical knowledge with computer use has been a preoccupation of ours for many years, beginning with projects with Logo in the 1980s. For example, learners' experience with mathematical variables can be transformed by first encountering them as "inputs" to Logo programs whose values can be easily changed with immediate observable effects; intensive quantities such as rates — notoriously difficult because of their abstractness — can be made "objects" and manipulated in many computational guises; conjectures can be tested by making constructions in dynamic geometry systems and transforming early encounters with proof from procedural exercises in validation to stimulating and exploratory exercises in explanation. This transformation of meanings generated in "contexts of integration", necessitates a conception of mathematical understanding by students, and of mathematical knowledge itself, that properly accounts for the specificity of situations and, most importantly, the contingencies of mathematical expression on tools and technologies and the communities in which they are used.

---

[1] An important exception is the work of French didactics on 'instrumental genesis'.

We coined the notion of *situated abstraction* to address these issues: it assumes as its starting point that tools and how they are used can be (if used fluently and mindfully) an integral part of an individual's evolving conceptualisation of mathematical knowledge, and that the activity – considered as a system with its rules of discourse – both shapes and is shaped by the tools (see Hoyles & Noss, 1992; Noss & Hoyles, 1996).

From a specifically mathematical point of view, a situated abstraction emerges as a means by which a community of – say – mathematical learners can develop a common discourse and agree with their teacher that they are talking about the same mathematical abstraction or set of abstractions. It does not matter whether or not the abstraction really *is* the same (indeed, it is a moot question as to what "the same" might mean in this context). Different meanings can be attached to the abstraction, but rather than being ignored, they can be explicitly brought into association and alignment by the teacher. Relatedly, the implication of this perspective is that students' expressions can gain mathematical legitimacy, *even if they differ substantially from traditional mathematical discourse,* and even if they are shaped and structured by the artefact in ways that lead them to diverge from established epistemologies.

References

Hoyles, C. & Noss, R. (1992) Looking back and looking forward. In: Hoyles C. & Noss R. (Eds.) *Learning Mathematics and Logo*. pp. 431-468. Cambridge: MIT Press.

Hoyles C. & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In: Bishop, A. (Ed.) *Second International Handbook of Research in Mathematics Education*. Dordrecht: Kluwer. page 323- 349.

Noss, R. & Hoyles, C. (1996) *Windows on Mathematical Meanings: Learning Cultures and Computers*. Dordrecht: Kluwer.

Undergraduate Mathematics Enhanced With Graphing Technology

[1]Rosihan M. Ali, and [2]Kor Liew Kee

[1]School of Mathematical Sciences, Universiti Sains Malaysia, Penang
[2]Faculty of Information Technology and Quantitative Sciences, MARA University of Technology, Kedah
[1]rosihan@cs.usm.my and [2] korlk@tm.net.my

Abstract. The School of Mathematical Sciences at Universiti Sains Malaysia offers a laboratory course on the integration of hand-held technology into the teaching and learning of mathematics since the beginning of the 2001/2002 academic year. This inquiry-based course highlights explorations and applications of mathematics in a data rich modeling environment. In addition, the course addresses several issues related to the effective integration of such technology into the mathematics curriculum. This paper discusses the appropriate use of graphing technology to present mathematical concepts and to support student's understanding in a student-centered learning environment, shares knowledge on the new mathematics that was made possible by the hand-held technology, and summarizes student reactions to the innovative learning mode.

1. Introduction

Research from the developed countries has shown that calculators can be used to enhance the understanding of mathematical concepts. It was reported that with proper guidance in using the calculator technology, students will not be using calculators unnecessarily for simple computations. Incidentally, the results from the Third International Mathematics and Science Study (TIMSS, 1999) showed that in most countries where emphasis on calculator uses was high, there was a positive association between calculator use and mathematics achievement. Since 1997 Universiti Sains Malaysia (USM) has developed an interest in graphic calculators, in particular, the CAS-enabled graphic calculator. The affordability, portability, and varying CAS capabilities of graphic calculators has led it to being taken up more widely as a tool to support and enhance the teaching and learning of senior secondary mathematics in countries such as Austria, Australia, Canada, Denmark, France, Germany and the United States. Beginning 2002 Singapore allows the use of non-CAS graphic calculators in its GCE A Level Exams in Further Mathematics. Over the last two years, the Ministry of Education has continuously supplied graphic calculators to several selected schools in Malaysia to explore the calculator efficacy in the teaching and learning of secondary mathematics.

The School of Mathematical Sciences at USM, in collaboration with colleagues from the School of Educational Studies, has developed from the ground up a calculator based laboratory course since the beginning of the 2001/2002 academic year. In this course, students are made acquainted with the capabilities of graphic calculator as an instructional tool. In addition, seminars addressing issues related to pedagogical and curricular changes driven by the integration of the new technology into the classroom are also called for in this course. The advent of technology has put at issue teaching pedagogy and strategies. A discussion on the constructivism perspectives that were implemented in the course could be found in Ali *et al.* (2002). This paper reflects on how graphing technology was used to present mathematical concepts and to support student's understanding in the course, now in its third year, shares knowledge on the new mathematics that was made possible by the technology, and summarizes student reactions to the innovative learning mode.

2. Course features

The laboratory course in graphic calculators seeks to explore the impact of such instructional devices and the perspectives they provide. The course is developed for pre-service teachers and students in mathematics. The course objectives are:

- To acquaint students with the CAS calculators and its capabilities.
- To understand the relevance of calculator technology in the teaching and learning of mathematics and sciences.
- To familiarize students with the issues involved in the use of calculator technology in the classroom.
- To model the effective integration of technology into the mathematics curriculum.
- To teach the development of data rich technology explorations that is designed around the capabilities of the calculators.

The course content includes topics from calculus, linear algebra, differential equations, and statistics. The TI-83Plus graphic calculator was used for statistics in the first half of the semester, while the TI-92Plus was used in the remaining weeks for calculus, linear algebra, and differential equations. Students were not required to purchase graphing calculators; each student had a calculator checked out for the duration of the course. There were 28 class meetings of about two hours. The primary teaching mode was an interactive lecture mode and in-class exploration activities alternately conducted. Class activities were supported with laboratory assignments that the students completed and turned in for assessment. The course culminated with a group project designed to foster students' knowledge and critical understanding of principles in mathematics and statistics.

3. The role of graphing technology

A graphic calculator is a powerful tool that can carry out complicated mathematical tasks, thus allowing students to spend more time to work with mathematics at a higher cognitive level. When used effectively, it becomes a tool to help students actively construct their own knowledge bases and skill sets. An important consideration in its use is made by Lim (2002), in that technology should not only affect *how* we teach because technology makes different approaches possible, technology should also change *what* we teach because some topics are made obsolete with technology while others are made possible with it.

Our course is developed around the capabilities of the technology to enhance the understanding and learning of mathematical concepts and theories. Particular attention was given in the design of laboratory explorations and scientific visualizations to ensure that the graphing technology plays a pivotal role in achieving the learning outcomes. Thus the same content cannot be done without the graphing technology.

The following examples illustrate how graphing technology was used in the course, the different instructional approaches adopted, and the new mathematics brought about by the technology.

Example 1. For the function $f(x) = x\sin\left(\frac{1}{x}\right)$, *find* $\lim_{x\to 0} f(x)$. *Investigate each limit graphically, numerically and symbolically.*

In this example, the multiple representation features of the graphic calculator are exploited to study the limit of functions. The graphical, tabular, and symbolic features of the calculator are incorporated to investigate the limit of $f(x) = x\sin(1/x)$ at $x = 0$. Although this is a typical problem in calculus, the oscillation nature of the graph around $x = 0$ is awfully difficult to visualize with chalk and board. The calculator easily overcomes this difficulty. Changing the window parameters on the graphing calculator allows the student the appropriate visualizations. In Figures 1-2, the student gets to see the oscillations around $x = 0$ as well as other properties of the graph such as symmetry. By using the trace feature to explore the functional values around $x = 0$, it is now an immediate step to deduce graphically that the limit is zero.

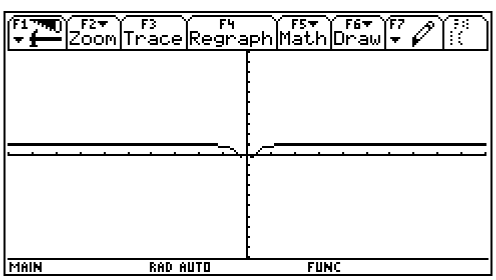


Figure 1. Graph of $f(x) = x\sin(1/x)$ on a standard viewing window

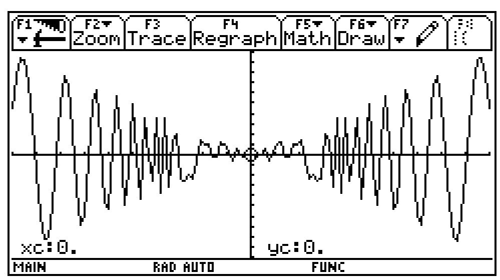
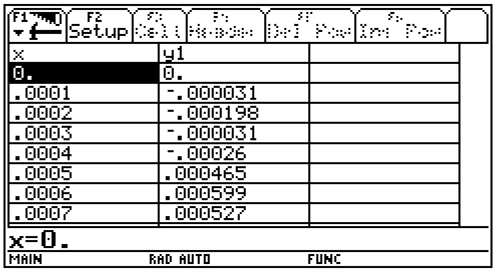


Figure 2. The oscillations of $f(x) = x\sin(1/x)$

The investigation of the limit is carried further numerically by creating a table of values. Scrolling through $x = 0$, the table values in Figure 3 supports the assertion that $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0.$

Finally the limit is found symbolically as illustrated in Figure 4.



x	y1
0.	0.
.0001	-.000031
.0002	-.000198
.0003	-.000031
.0004	-.00026
.0005	.000465
.0006	.000599
.0007	.000527

Figure 3. Table of values for $f(x) = x\sin(1/x)$

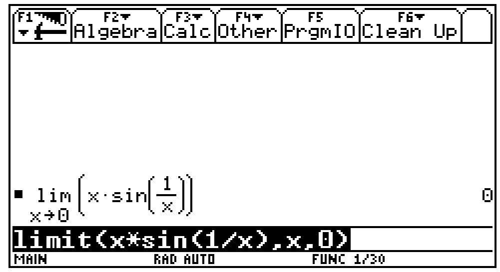


Figure 4. The limit of $f(x) = x\sin(1/x)$ at $x = 0$

It is very important to note that the above efforts are tantamount to an illustration of a result, not a proof. The graphical device gives the student the confidence that the answer is correct, however, the result must be proven rigorously before we may infer it to be true. This is established by an application of the “sandwich” or “squeeze” principle.

Example 2. Let $u_1 = \frac{1}{4}$, $u_n = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{4\cdot 6\cdot 8\cdots(2n+2)} = \frac{2n-1}{2n+2}u_{n-1}$, $n \geq 2$. *Investigate* $\lim_{n\to\infty} u_n$.

This is an example on an application of the Bounded Monotonic Convergence Theorem (BMCT) for a sequence of terms. The sequence $\{u_n\}$ defined recursively is first investigated graphically (Figures 5-6) and numerically (Figure 7).

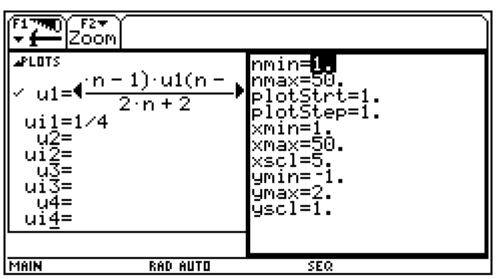


Figure 5. A split screen of two editors

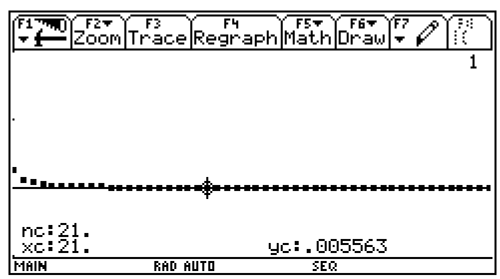


Figure 6. Graph of the sequence $\{u_n\}$.

From the visualization and by using the trace feature, the graph suggests that

- $u_n \geq 0$,
- $\{u_n\}$ is decreasing, and
- $\lim_{n\to\infty} u_n = 0$.

The table of values also supports these observations.

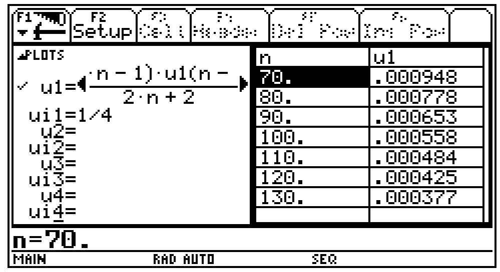


Figure 7. Table of values for the sequence $\{u_n\}$

The explorations give valuable information about the sequence $\{u_n\}$. It gives the students concrete feedback about the accuracy of their ideas, and draws them to apply the BMCT in proving the existence of the limit. However the assertion that the limit is zero is still a formidable task to show since it cannot be obtained from the BMCT. The student is required to use the Raabe's test for a positive-term series to establish this fact.

Example 3. The economic performance of a country may be measured by its gross domestic product GDP. The outcome variable GDP depends on several components. Table 1 provides data on the Malaysian GDP (Y) and the explanatory variables - growth in GNP X_1, *agricultural growth* X_2, *industrial growth* X_3, *population growth* X_4, *export growth* X_5, *and import growth* X_6. *Assuming the regression function of* Y *on* $X_1, X_2, X_3, X_4, X_5, X_6$ *is linear, find the estimated regression function.*

Table 1. Economic performance of Malaysia from 1985 – 1997

Item Number	GDP Y (RM Millions)	Growth GNP X_1	Agri Growth X_2	Ind Growth X_3	Pop Growth X_4	Exp Growth X_5	Import Growth X_6
1985	29280	4.5	4.4	8.7	2.0	4.9	7.3
1986	29280	4.5	4.2	8.7	2.1	7.5	8.9
1987	31270	4.4	3.0	6.7	1.9	10.7	6.4
1988	27580	4.3	3.0	6.0	1.9	10.2	5.2
1989	31230	4.1	3.4	5.8	2.2	9.7	-0.7
1990	34680	4.0	3.7	6.1	2.2	9.4	0.4
1991	37480	4.0	3.9	6.5	2.2	9.8	3.7
1992	42400	4.0	3.8	7.1	2.3	10.3	5.6
1993	46980	2.9	3.7	7.7	2.2	10.9	7.2
1994	57568	3.2	3.6	8.0	2.0	11.3	7.9
1996	70626	5.6	2.8	9.8	2.4	17.8	15.7
1997	85311	5.7	2.6	11.0	2.4	17.8	15.7

Source: World Development Report, Oxford University Press.
Missing observations for the year 1995

This exploration applies matrix arithmetic to obtain the parameter estimates by the *principle of least squares.* Mathematically, we seek the coefficient vector U satisfying $AU = Y$, where

$$A = \begin{bmatrix} 1 & 4.5 & 4.4 & \cdots & 7.3 \\ 1 & 4.5 & 4.2 & \cdots & 8.9 \\ 1 & 4.4 & 3.0 & \cdots & 6.4 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 3.2 & 3.6 & \cdots & 7.9 \\ 1 & 5.6 & 2.8 & \cdots & 15.7 \\ 1 & 5.7 & 2.6 & \cdots & 15.7 \end{bmatrix}, \quad U = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_6 \\ \hat{\beta}_7 \end{pmatrix}, \quad Y = \begin{pmatrix} 29280 \\ 29280 \\ 31270 \\ \vdots \\ 57568 \\ 70626 \\ 85311 \end{pmatrix}.$$

Since A is not a square matrix, the defining matrix equation $AU = Y$ is first transformed into an equivalent form $\left(A^T A\right)U = A^T Y$. The solution U is obtained from the equation $U = \left(A^T A\right)^{-1}\left(A^T Y\right)$.

Matrix notation and arithmetic is invaluable in finding parameter estimates for multiple linear regressions. Clearly the computations involved are tedious and cumbersome, and problems of this nature are normally avoided in class. However a graphic calculator is a very useful device to make these computations as illustrated in the Figures 8-10 below.

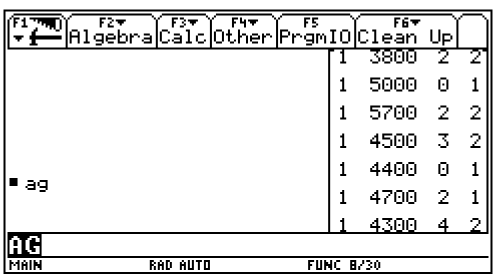


Figure 8. The entries of the matrix A.

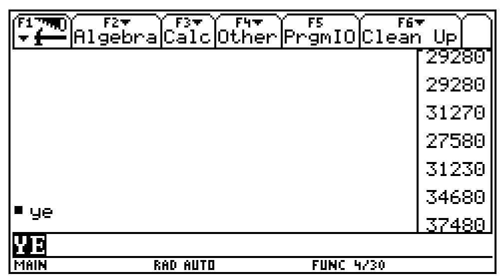


Figure 9. The vector Y.

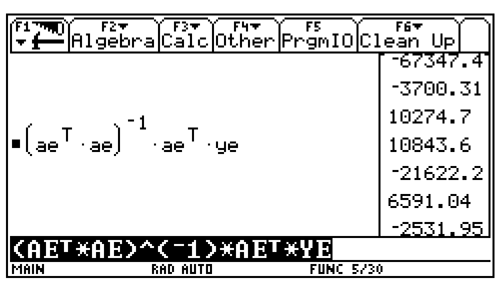


Figure 10. The solution of the coefficient vector U.

Thus the estimated regression function is

$$\hat{\mu}_Y(x_1, x_2, x_3, x_4, x_5, x_6) = -67347.4 - 3700.31x_1 + 10274.7x_2 + 10843.6x_3 - 21622.2x_4 + 6591.04x_5 - 2531.95x_6.$$

This regression function provides a useful model for forecasting the economic performance. Further analysis may be made to arrive at the conclusions that industrial growth is positively associated $(p < 0.02)$, as well as growth in export $(p < 0.04)$ and growth in import $(p < 0.10)$.

Polynomial regression models are of course special cases of multiple linear regression functions. As a further example, students were provided with a two-variable real data and were asked to find the linear and quadratic regression models from the least squares best fit method above. These models were next compared with the regression equations obtained from the calculator's *Stats Editor*. The equations found were exactly identical. The students now have an enhanced understanding of regression equations obtained from an application of statistics software.

Example 4. In this example, students retrieved from a www.census.gov website data on the population of Malaysia from 1950 – 2001. The learning objectives are to apply the concept of discrete differential data to formulate a logistic population growth model

$$\frac{dP}{dt} = kP(C - P), \quad P(0) = 6{,}433{,}799,$$

where C is the carrying capacity, and to make predictions. Data can be transmitted from one calculator to the others by using a TI-92Plus cable, thus not everyone needs to input the data. Figure 11 gives a scatter plot of the population data.

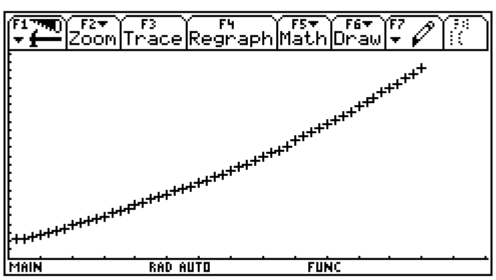


Figure 11. A scatter plot of the Malaysian population from 1950 – 2001

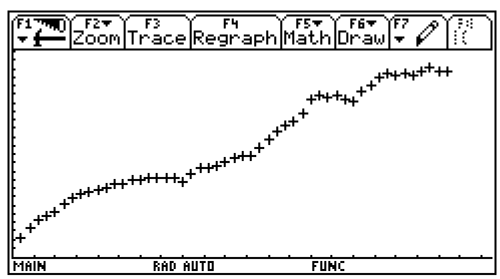


Figure 12. A scatter plot of the discrete derivative

The equation above can be expressed in the form

$$\frac{1}{P}\frac{dP}{dt} = -kP + kC := ax + b \qquad (1)$$

with $x = P.$ The discrete derivatives

$$P'(t_{n-1}) = \frac{P(t_n) - P(t_{n-1})}{t_n - t_{n-1}},$$

are computed (Figure 12). Equation (1) requires us to compute the points $\left(P_n, \frac{P'_n}{P_n}\right)$ with $P_n = P(t_n),$ and the best linear regression equation to this data is next obtained from the *Stats Editor* (Figures 13 and 14).

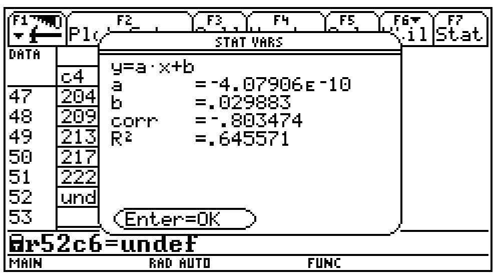


Figure 13. The linear regression equation

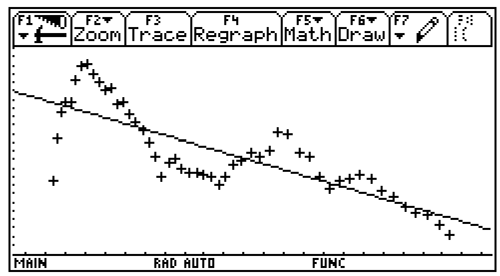


Figure 14. A scatter plot of population growth and the regression line

Equation (1) shows that the linear regression line models the differential equation $\frac{dP}{dt} = aP^2 + bP.$ Subject to an initial condition $P(0),$ the differential equation solver *deSolve* (Figure 15) easily determines the population model

$$P(t) = \frac{7.32584E7 \cdot (1.03033)^t}{(1.03033)^t + 10.3865}.$$

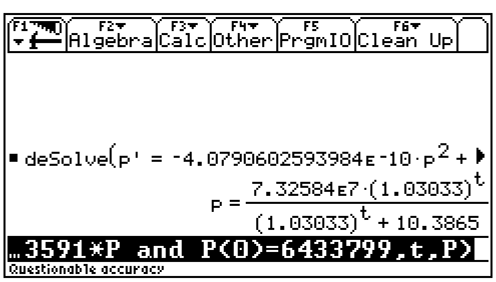


Figure 15. A symbolic solution of the population model

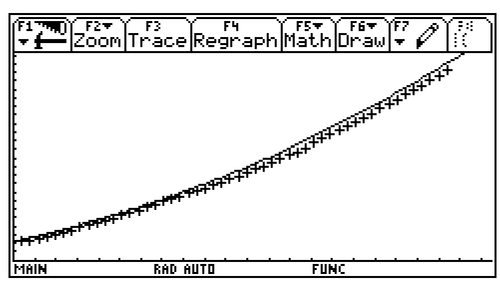


Figure 16. Graphs of the population scatter plot and the population model

The population model $P(t)$ and the population scatter plot are graphed on the same graph in Figure 16. The model gives a very good fit to the population data.

The graph of the population model is next plotted over a period of 400 years in Figure 17. Predictions can be made; for instance, the population of Malaysia in 2020 will be $P(70) = 32.09$ *million.* In addition, the carrying capacity is seen to be about 73 million.

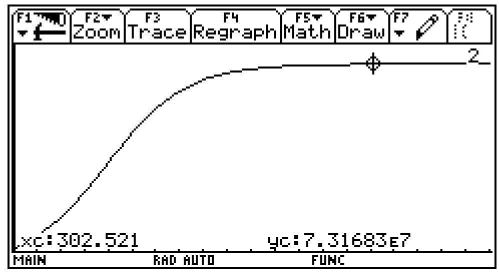


Figure 17. Graph of the population model

It is clear from the examples above that the graphic calculator allows student learning to occur at a higher cognitive level and serves to facilitate inquiries, explorations, and problem-solving activities. Our course put emphasis on its use as

- a tool for the symbolic manipulation or graphical display of mathematical functions and equations,
- a facility for the collection, examination and analysis of data,
- a tool to foster collaborative learning and teach students to work as a team,
- a tool to aid in solving realistic problems that enables the student to concentrate on problem aspects and interpretation rather than computational aspects, and
- a tool to discover, visualize, or investigate mathematical theories.

In this student-centered learning environment, the graphic calculator encourages students to reflect on and elaborate not just their own ideas, but those of their peers as well.

4. Survey summary

To monitor the impact of graphic calculators (GC) in the course, a semi-structured survey was prepared and implemented upon completion of the course. This survey requested information on students' perception of their understanding and impression of the course taught with GC, and also sought their views on the educational value of integrating the GC into mathematics in general. The four main aspects observed in this study are the *cognitive* domain including *tool competency*, the *affective* domain, *behavioral* domain and the *value* domain.

Quantitative data

The instrument used in this study was a survey questionnaire made up of 53 items. Item 1 to item 11 (11 items) measure the confidence in the learning of mathematics with GC (MatGC) and item 12 to item 23 (12 items) measure the confidence in using GC (ConfGC). These items were adapted from an instrument called the Attitudes to Technology in Mathematics

Learning Questionnaire (Mtech) developed and validated by Fogarty *et al.* (2001). Fogarty *et al.* reported an internal consistency with Cronbach alpha value of .90 for both the confidence in learning mathematics with GC and confidence in using GC respectively.

The overall breakdown of the items in the instrument of this study is grouped into five categories or aspects. There are 22 items in the *cognitive* category measuring students' intellectual knowledge applied to the learning of the lab course, 5 items in the *tool* competency gathered to measure the required skills in using GC in the lab course, 11 items in the *affect* category concentrating on the students' positive and negative feelings concerning the application of GC technology in the lab course, 12 items on *value* measuring the use, relevance and worth of GC in personal and professional career. Finally there are 3 items in the *behavioural* category measuring the nature of activity (grouped or activity-oriented) in the process of learning. All items are 5-point Likert scale ranging from 1 (strongly disagree) through 3 (neutral) to 5 (strongly agree). A mean score of more than 3.0000 indicates a favorable response towards the usage of GC in learning the lab course. The questionnaires were administered to the entire population (N = 39) at the end of the lab course.

Table 2. A summary of the items according to the categories in the survey questionnaire

Category/aspect	Related Items
Cognitive	1,7,10,11,24,25,26,27,28,29,30,33,34,37,38,39,40,43,47,49,51,53
Tool competency	12,13,14,15
Affect	8,9,16,17,18,19,20,21,22,23,45
Value	2,3,4,5,6,31,35,36,46,48,50,52
Behaviour	41,42,44

Table 3. Comparing means from the different aspects observed in the study

Aspects	N	Mean	Std. Deviation
Cognitive	39	3.7155	.57296
Tool competency	39	2.4718	1.05978
Affect	39	3.5000	.77040
Behaviour	39	3.1197	1.03304
Value	39	3.4190	.80116
MatGC	39	3.6981	.60589
ConfGC	39	3.2168	.89477

Table 4. Comparing gender mean between the different aspects observed in the study

Gender		Cognitive	Tool	Affect	Behaviour	Value	MatGC	ConfGC
Male	Mean	3.6167	2.4000	3.4697	3.4444	3.0000	3.4848	3.1970
	N	6	6	6	6	6	6	6
	Std.dev.	.50563	1.10272	.82204	1.12875	1.17851	.75551	.80989
Female	Mean	3.7335	2.4848	3.5055	3.0606	3.4952	3.7369	3.2204
	N	33	33	33	33	33	33	33
	Std.dev.	.58968	1.06891	.77396	1.02217	.71154	.58023	.92025
Total	Mean	3.7155	2.4718	3.5000	3.1197	3.4190	3.6867	3.2015
	N	39	39	39	39	39	39	39
	Std.dev	.57296	1.05978	.77040	1.03304	.80116	.58221	.87673

Table 5. Comparing program mean between the different aspects observed in the study

Program		Cognitive	Tool	Affective	Behavioural	Value	MatGC	ConfGC
B.Sc.	Mean	3.7680	2.6000	3.5692	2.9167	3.4943	3.7461	3.3182
	N	28	28	28	28	28	28	28
	Std.dev.	.56812	.98583	.76809	1.10601	.76661	.62020	.93351
B.Sc.Ed.	Mean	3.5444	2.0000	3.2343	3.5926	3.1111	3.5020	2.8384
	N	9	9	9	9	9	9	9
	Std.dev.	.60386	1.16619	.68487	.59577	.90906	.42010	.56732
Total	Mean	3.7137	2.4541	3.4877	3.0811	3.4011	3.6867	3.2015
	N	37	37	37	37	37	37	37
	Std.dev	.57668	1.04844	.75360	1.04055	.80759	.58221	.87673

Discussion

Using SPSS, the quantitative data were analyzed. The reliability coefficient alpha was .6032 for MatGC and .7460 for ConfGC. Nonetheless, the reliability coefficient alpha obtained from pilot study was .7484 for MatGC and .7740 for ConfGC showing that the items have a good internal consistency that is reliable. Tables 3 – 5 show the results of the data analyses.

All means recorded for the five major aspects (Table 3) including the confidence in using GC to learn mathematics (MatGC) and confidence in using GC (ConfGC) except for tool competency scored higher than 3.0000 indicating that students favoured the introduction of GC in the lab course. Tool competency managed only with a mean score of 2.4718 showing that students still lack some of the required skills in using GC to learn the lab course efficiently and effectively.

The gender mean in Table 4 shows that the female students scored generally higher than the male students in all the aspects observed. The overall score for the cognitive domain (mean = 3.7155) is the highest followed by the MatGC (mean = 3.6867).

The program mean in Table 5 displays a favorable response on the usage of GC except for the *tool* competency which scored only a mean of 2.4541. The Bachelor of Science (B.Sc.) program scored low in the behaviour domain (mean = 2.9167) showing that the students in this program preferred to study individually as compared to those in the Bachelor of Science with Education (B.Sc. Ed., mean = 3.5926). In the aspect on confidence in using GC (ConfGC), the Bachelor of Science with Education students scored only a mean of 2.8384 which was below the average of 3.0000 indicating that these students were not confident enough in handling the GC in general.

The overall response shown by the students towards the graphing technology in teaching the lab course is positive and favorable. The tool competency scored the lowest among all means and an interview with the instructor of the lab course confirmed that this is true because students were consistently force to learn and use new GC commands when switching from topic to topic during the whole course. As a result, many students might be confused or not able to recall the specific GC commands as fast as they should in the process of solving a problem without any reference. Such inadequacy eventually slowed down the whole process of problem solving and is likely to be the main cause of frustrations among the students.

Qualitative data

The qualitative data were obtained from the four open-ended questions accompanying the survey questionnaires. The four questions asked were

- What do you think are the benefits of GCs in learning mathematics?
- Do you think GC is a useful tool to learn mathematics? Why?
- What is your opinion of mathematics now after you are exposed to the use of GCs to learn mathematics?
- Would you prefer to be a person with the knowledge of GC? Why?

The results of a content analysis found that majority of the students have positive opinions towards learning mathematics with GC. These opinions can be classified into three major aspects such as *cognitive competence*, *affect*, and *value*. The comments commonly given by the students to the above questions were GC helps to save time in solving a problem, allows more exploration than traditional method, tests many concepts in a shorter time, arouses interest in learning mathematics in general, reduces careless mistakes, makes mathematics easier to learn, gives more accurate answer and is important to the future generation and career.

Below are some of the interesting comments grouped according to the three major aspects made by the students:

Cognitive competence

Cognitive competence is associated with the students' ability to learn mathematics with GC (*cognitive* domain) and also their skills in handling the GC (*tool* competence). Their opinions on the advantages and disadvantages of using GC in the learning of mathematics were analyzed. The following are examples of the comments:

Student 2: "*… GC helps to solve difficult questions where normal people cannot do so in a short period.*"

Student 12: "*GC gives simulation…Something that you have never imagine before…No difficult programming (e.g. MATLAB)…GC enhanced my learning of linear algebra, calculus, differential equation and statistics…I used to have difficulties in imagining statistical inference, normal distribution etc. With GC, everything can now be seen right in front of my eyes.*"

Student 22: "*With GC, we (students) can achieve a higher level of learning in math at an earlier stage rather than just memorizing formula like what we usually do.*"

Student 25: "*GC provides answers graphically and numerically. Given an equation, the graph is plotted itself with the right commands and scales. This has made me realized that basic calculus could be easily done with the help of GC since I do have a shaky basis… GC helps students to explore a question from different angles. We can play with the data and can reuse them again and again…*"

Student 26: "*GC can handle big data that may be confusing in manual calculation.*"

Student 30: "*… the ideas/concepts of math used to be very abstract and caused me to lost my interest in learning math further…But now, with GC I am able to visualize the concepts better…*"

Affect

The term *affect* looks into how students felt about the lab course taught with GC, enjoyed the course, or were stressed by and daunted by the introduction of the new tool in the mathematics curriculum. For example:

Student 12: "*GC is the best device… With GC I am now confident to tackle any math problem on my own…I am lucky to know and have a chance to learn the GC technology.*"

Student 29: "*GC helps to alleviate my drudgery towards math…math is more exciting now compare to before the introduction of GC.*"

Student 30: "*Math is easier now as learning concepts and ideas are clearer with GC. Before this I was afraid of math because it was difficult.*"

Student 37: "*…I am happy because I know and can use more methods to solve a mathematics problem with GC…*"

Value

Value is associated with the importance of graphing technology in relation to students' daily life, future career, and professional job. Examples of students' comments are:

Student 1: "*GC is an improvement in math gadget and I want to be one of those who know about GC…*"

Student 7: " *…GC makes me think analytically … teaching math with GC emphasizes more on interpretation skills in solving problems…these skills are useful to my future career.*"

Student 22: "*We can appreciate math better with the use of GC. In the traditional methods of learning math, sometimes the beauty of math is not enhanced…with GC, we look forward to explore the subject and integrate it with other fields.*"

Student 23: "*With the knowledge of GC, I can appreciate Linear Algebra more after attending the Lab Course.*"

Student 25: "*…since we are moving towards the technology era, I would not want to miss my chance of getting my hands on this sophisticated tool. It could be of great use in the future.*"

Student 29: "*…I prefer to use technology when performing my daily tasks*…"

Student 31: "*… with much guidance, students will be able to learn how to interpret mathematical results in real life.*"

Student 39: "*If I have enough money, I will buy a GC and use it to enhance my understanding of mathematical concepts. I can now understand better the concepts in algebra, statistics, calculus, and differential equations… I can become a professional teacher with the knowledge of GC.*"

While some of the neutral comments are:

Student 16: *"I don't think GC is a useful tool because student doesn't know the traditional way of calculation by just referring to GC... but it will be useful if the traditional method of solving the problem proves to be too difficult to do by hand."*

Student 36: "*GC is only useful if we have a trainer to teach the teacher to use it*."

Discussion

The qualitative results showed an overall favorable attitude towards the use of GC technology in the lab course. The students' positive response on visualization and exploration coincides with Scariano and Calzada's (1994) assertion that GC "enhances visualization and invites self-discovery" (p.61). The expressions illustrated in the affect domain showed that students were highly motivated and excited about the idea of introducing GC technology in the lab course. This result is supported by Stick (1997) who observed that university students were captivated with the visual displays and were highly motivated throughout the semester. Similarly the feedback from Milou's (1999) study on the teachers also concluded that calculators help to motivate student's learning. He reported further that using calculators can increase the level of achievement and understanding in algebra and precalculus. Other comment such as GC allows students to focus on concepts and brings real world data into the classroom were also reported by Martinez-Cruz and Ratliff (1998).

5. Concluding remarks

We are very encouraged with the survey findings, which showed that the majority of students responded positively and favorably towards undergraduate mathematics enhanced with graphing technology. Our efforts in making changes to a curriculum impacted by graphing technology and the adoption of the new pedagogical practice have been rewarding. During the course, students were seldom seen to work alone but tutoring each other became a common sight. Evidently technology promotes student's motivation, inspires critical thinking and improves problem-solving skills. Our next initiative is to incorporate graphing technology into other courses so as to enable students the continued familiarity in using the most advanced features of the graphing technology. We also need to face the priority challenge of developing appropriate and valid test items for the purpose of classroom assessment in this hand-held technology enabled classroom-learning environment.

References

Ali, R.M., Seth, D.L., Zainuddin, Z., Kassim, S., Sulaiman, H., & Haili, H.K. (2002). Learning and teaching mathematics with a graphic calculator. *Bulletin Malaysian Math. Sc. Soc. (Vol. 25, pp. 53-82).*

Forgaty, G., Cretchley, P., Harman, C., Ellerton, N., & Konki, N. (2001). Validation of a questionnaire to measure mathematics confidence, computer confidence, and attitudes towards the use of technology for learning mathematics. *Mathematics Education Research Journal*, *13*(2), 154-160.

Lim, S.K.(2002). Mathematics education within the formal Singapore education system: Where do we go from here? In D. Edge & Yeap B.H. (Eds.), *Proc. Second East Asia Regional Conf. On Math. Education (Vol. 1, pp.29-37).* Assoc. Math. Educators: Singapore.

Martinez-Cruz, A. M. & Ratliff, M. I.(1998). Beyond modeling world records with a graphing calculator: Assessing the appropriateness of models. *Mathematics and Computer Education*, pp. 576-581.

Milou, E. (1999). The graphing calculator: A survey of classroom usage. *School Science and Mathematics*, 99(3), pp. 133-140.

Scariano, S. M. & Calzada, M. E. (1994). The role of the graphing calculator in the basic skills mathematics curriculum. *Mathematics and Computer Education*, pp. 60-69.

Stick, M. E. (1997). Calculus reform and graphing calculators: A university view. *Mathematics Teacher*, pp. 356-360.